Lecture #4

- Conspiracy in the Take-Grant Protection Model
- *de facto* rules (information flow)
- Knowing in a combined graph
- Basics of Schematic Protection Model

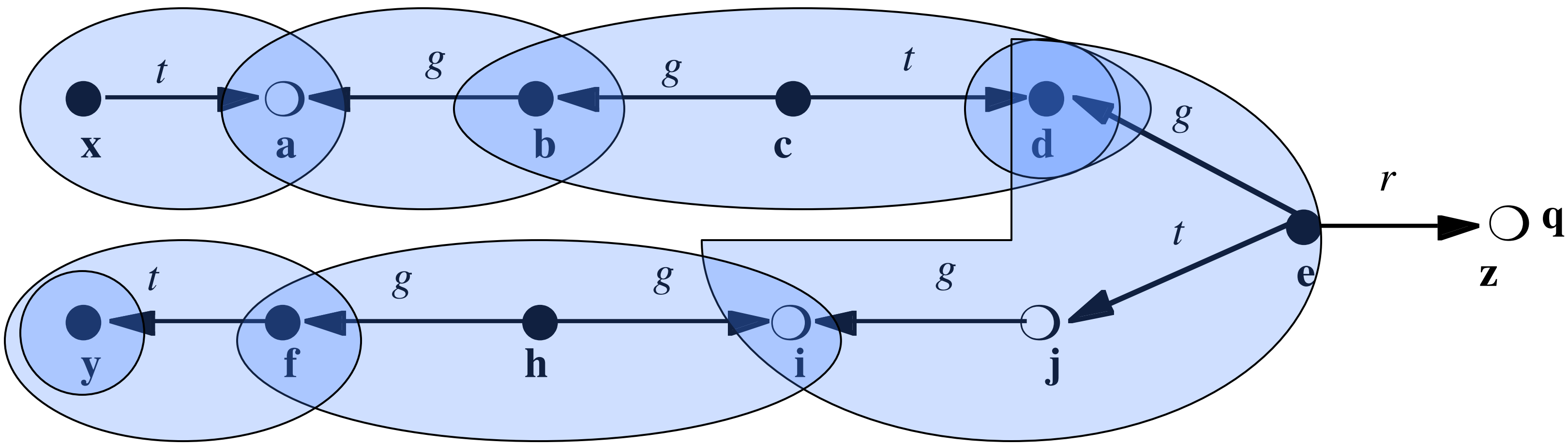
Conspiracy

- Minimum number of actors to generate a witness for *can•share*(α, **x**, **y**, G_0)
  - Actor is defined as **x** such that **x** initiates ϱ_i
- Access set describes the "reach" of a subject
- Deletion set is set of vertices that cannot be involved in a transfer of rights
- Build *conspiracy graph* to capture how rights flow, and derive actors from it

Access Set

- *Access set A(**y**) with focus* **y**: set of vertices:
  - { **y** }
  - { **x** | **y** initially spans to **x** }
  - { **x** | **y** terminally spans to **x** }
- Idea is that focus can give rights to, or acquire rights from, a vertex in this set

Example



- $A(\mathbf{x}) = \{\ \mathbf{x}, \mathbf{a}\ \}$
- $A(\mathbf{b}) = \{\ \mathbf{b}, \mathbf{a}\ \}$
- $A(\mathbf{c}) = \{\ \mathbf{c}, \mathbf{b}, \mathbf{d}\ \}$
- $A(\mathbf{d}) = \{\ \mathbf{d}\ \}$
- $A(\mathbf{e}) = \{\ \mathbf{e}, \mathbf{d}, \mathbf{i}, \mathbf{j}\ \}$
- $A(\mathbf{h}) = \{\ \mathbf{h}, \mathbf{f}, \mathbf{i}\ \}$
- $A(\mathbf{f}) = \{\ \mathbf{f}, \mathbf{y}\ \}$
- $A(\mathbf{y}) = \{\ \mathbf{y}\ \}$

tg-sink

- $\mathbf{x_0}$, only incoming t edge
- $\mathbf{x}_i$, two incoming incident edges, both labeled t or both labeled g
- $\mathbf{x}_n$, only incoming g edge

Necessity

- Lower bound on number of conspirators
  - Rights can be transmitted to any vertex in the access set
  - Rights can be "passed along" through the overlap of access sets, *unless* common vertex cannot initiate rule (*tg*-sink)
  - If only common vertex is *tg*-sink, must aid in transfer

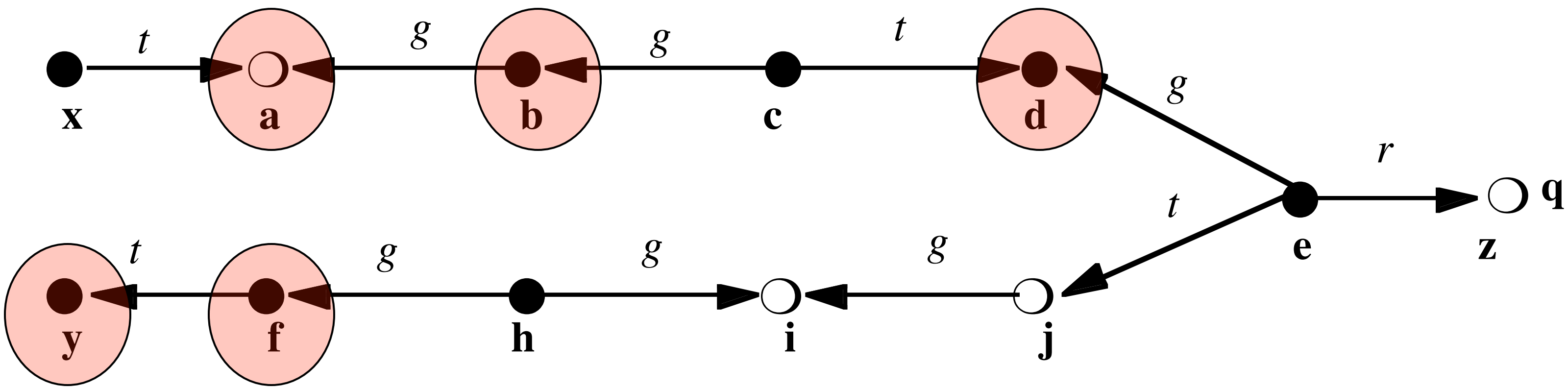
Necessity Theorem

- Let *can•share*(α, **p**, **q**, G) hold, and define G_0 to be G–{ **q** }. Let k be the number of access sets in a minimal cover of G_0, and let l be the number of *tg*-sinks. Then $k + l$ initiators are necessary to witness *can•share*(α, **p**, **q**, G).

Deletion Set

- Deletion set $\delta(\mathbf{y}, \mathbf{y}')$: contains those vertices in $A(\mathbf{y}) \cap A(\mathbf{y}')$ such that:
  - $\mathbf{y}$ initially spans to $\mathbf{z}$ and $\mathbf{y}'$ terminally spans to $\mathbf{z}$;
  - $\mathbf{y}$ terminally spans to $\mathbf{z}$ and $\mathbf{y}'$ initially spans to $\mathbf{z}$;
  - $\mathbf{z} = \mathbf{y}$
  - $\mathbf{z} = \mathbf{y}'$
- Idea is that rights can be transferred between $\mathbf{y}$ and $\mathbf{y}'$ if this set non-empty

Example



- $\delta(\mathbf{x}, \mathbf{b}) = \{ \mathbf{a} \}$
- $\delta(\mathbf{b}, \mathbf{c}) = \{ \mathbf{b} \}$
- $\delta(\mathbf{c}, \mathbf{d}) = \{ \mathbf{d} \}$
- $\delta(\mathbf{c}, \mathbf{e}) = \{ \mathbf{d} \}$
- $\delta(\mathbf{d}, \mathbf{e}) = \{ \mathbf{d} \}$
- $\delta(\mathbf{y}, \mathbf{f}) = \{ \mathbf{y} \}$
- $\delta(\mathbf{h}, \mathbf{f}) = \{ \mathbf{f} \}$
- $\delta(\mathbf{e}, \mathbf{h}) = \varnothing$

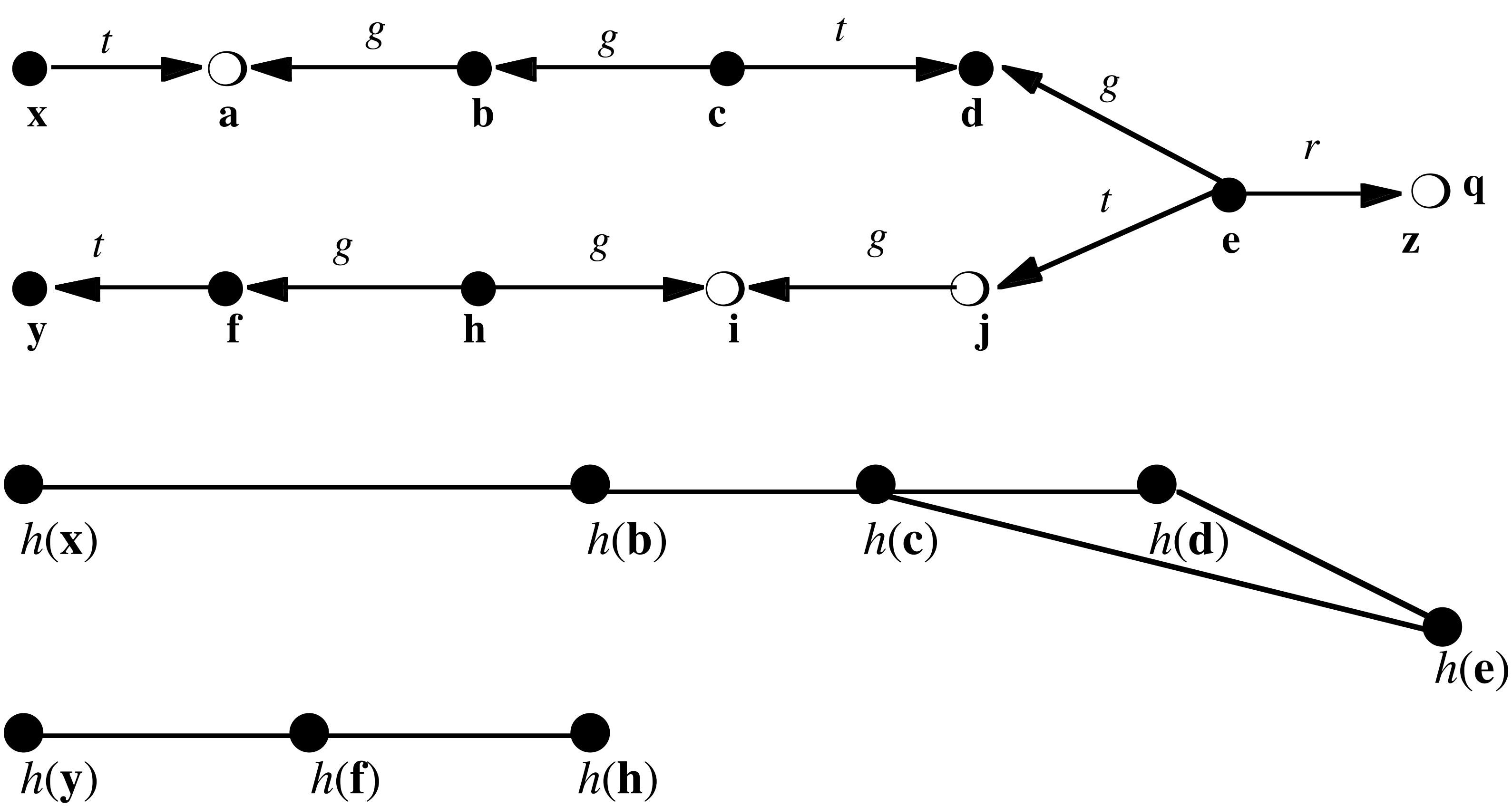
Sufficiency

- Consider $A(\mathbf{x}_i) \cap A(\mathbf{x}_{i+1}) = \{ \mathbf{y} \}$
  - If edges incoming to **y** are *both t* or *both g*, **y** must act
  - If edges incoming to **y** are *t* and *g*, it's a bridge and **y** need not act
- So, in first case, need one additional operation initiated by **y**
- Note: **y** is a *tg*-sink in these cases

Conspiracy Graph

- Abstracted graph H from G_0:
  - Each subject $\mathbf{x} \in G_0$ corresponds to a vertex $h(\mathbf{x}) \in H$
  - If $\delta(\mathbf{x}, \mathbf{y}) \neq \varnothing$, there is an edge between $h(\mathbf{x})$ and $h(\mathbf{y})$ in H
- Idea is that if $h(\mathbf{x})$, $h(\mathbf{y})$ are connected in H, then rights can be transferred between $\mathbf{x}$ and $\mathbf{y}$ in G_0

Example



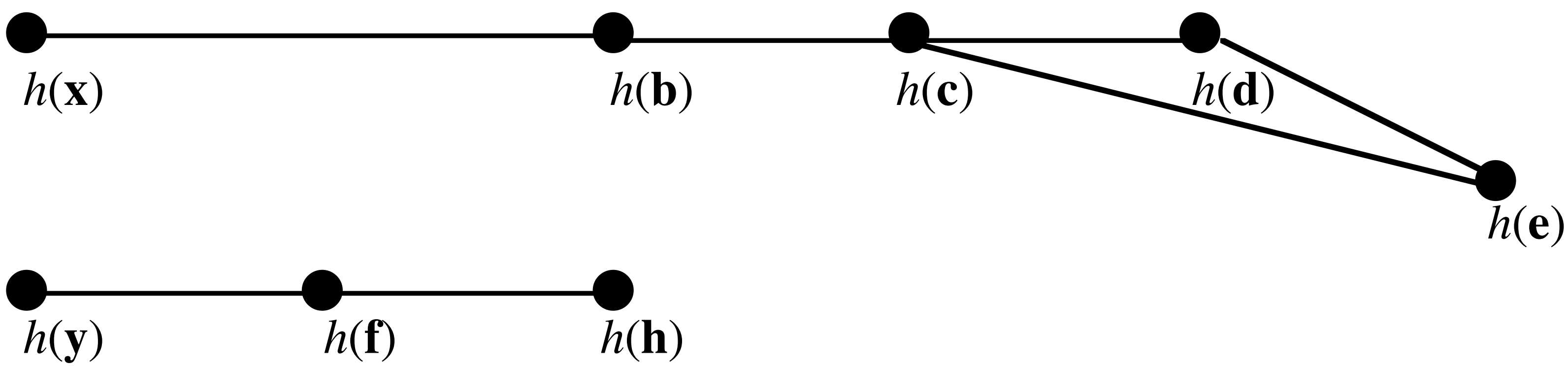
Sharing

- $I(\mathbf{x})$: $h(\mathbf{x})$, all vertices $h(\mathbf{y})$ such that **y** initially spans to **x**
- $T(\mathbf{x})$: $h(\mathbf{x})$, all vertices $h(\mathbf{y})$ such that **y** terminally spans to **x**
- Theorem: *can•share*$(\alpha, \mathbf{x}, \mathbf{y}, G_0)$ iff there exists a path from some $h(\mathbf{p})$ in $I(\mathbf{x})$ to some $h(\mathbf{q})$ in $T(\mathbf{y})$
  - Idea: path exists if access sets overlap and rights can be transferred between endpoints
  - Note *tg*-sinks correspond to singleton access sets with foci that must act (idea of deletion sets)

Counting Conspirators

- Theorem: if there are l vertices on shortest path between $h(\mathbf{p}), h(\mathbf{q})$ in above theorem, l conspirators necessary and sufficient to witness
  - Follows immediately from previous two theorems, definitions

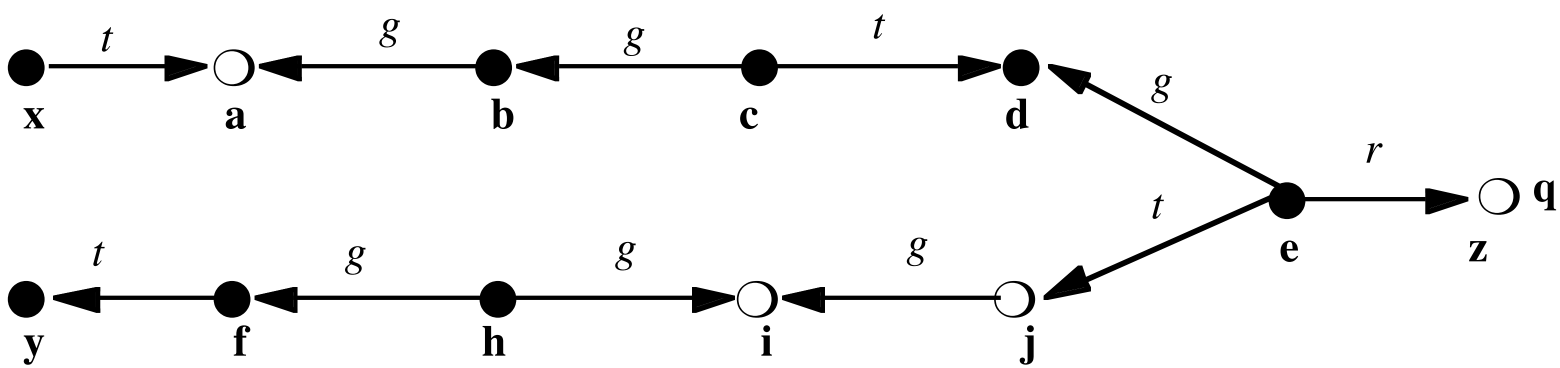
Example: Conspirators



- $I(\mathbf{x}) = \{ h(\mathbf{x}) \}$, $T(\mathbf{z}) = \{ h(\mathbf{e}) \}$
- Path between $h(\mathbf{x})$, $h(\mathbf{e})$ so $can\bullet share(r, \mathbf{x}, \mathbf{z}, G_0)$
- Shortest path between $h(\mathbf{x})$, $h(\mathbf{e})$ has 4 vertices

$\Rightarrow$ Conspirators are **e**, **c**, **b**, **x**

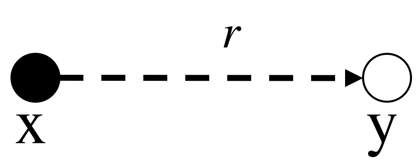
Example: Witness



- **e** grants (*r* to **z**) to **d**
- **c** takes (*r* to **z**) from **d**
- **c** grants (*r* to **z**) to **b**
- **b** grants (*r* to **z**) to **a**
- **x** takes (*r* to **z**) from **a**

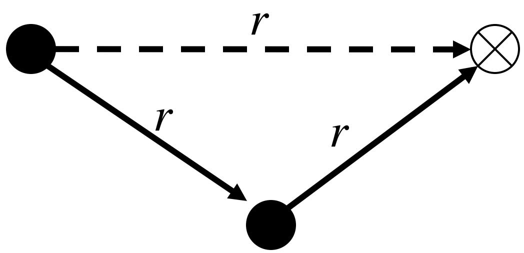
de facto Rules

- These deal with information flow
- Not graph rewriting rules
  – Add no edges
  – Instead, represent flows by “implicit” edges, shown as:

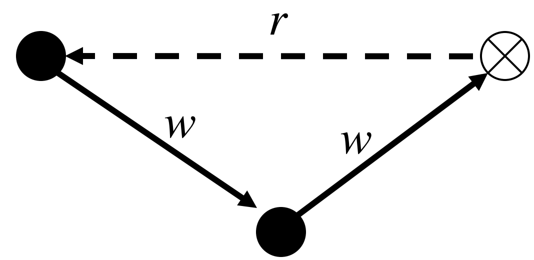


Rules

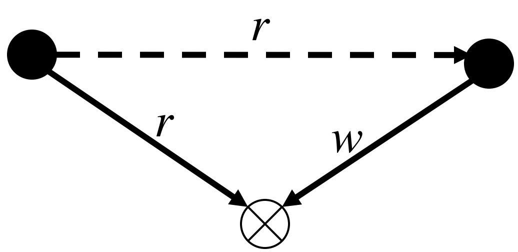
spy



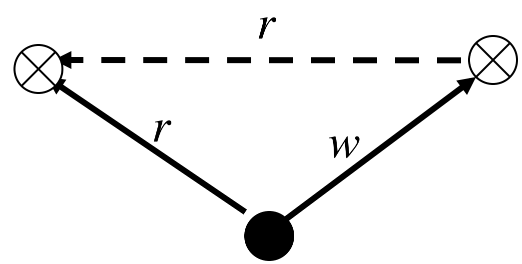
find



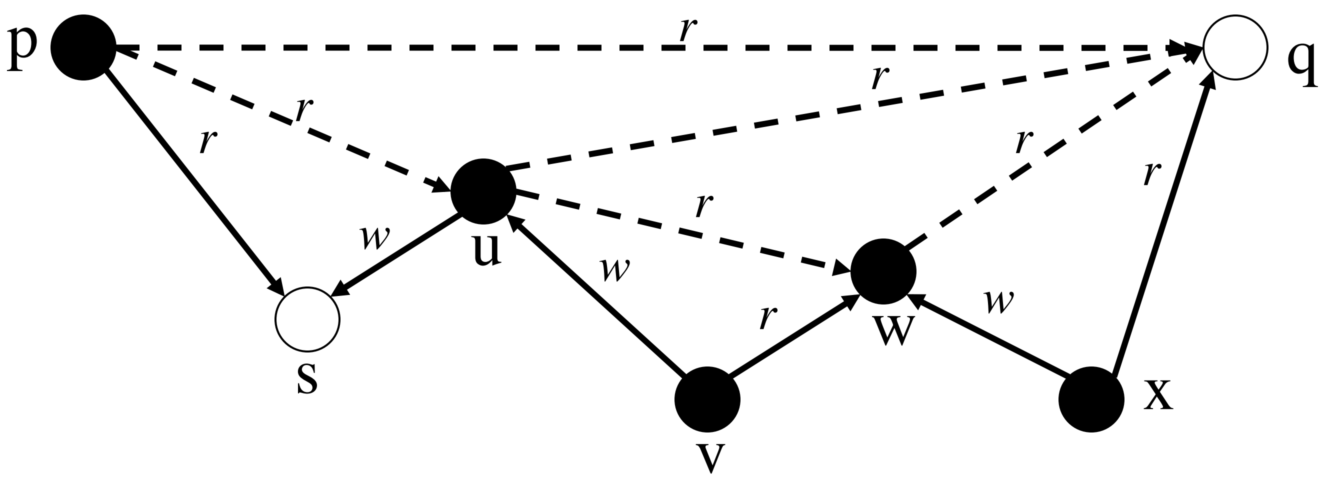
post



pass



Example



u posts through **s** to **p**

v passes from **w** to **u**

w spies through **x** to **q**

u spies through **w** to **q**

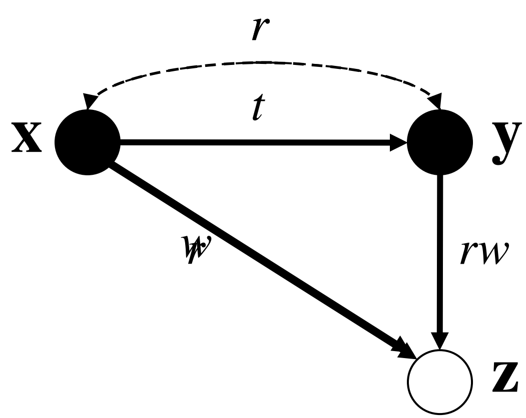
p spies through **u** to **q**

can•know

Definition:

- *can•know*(**x**, **y**, G_0) if, and only if, there is a sequence of protection graphs $G_0, \ldots, G_n$ such that $G_0 \vdash^* G_n$ using *de jure* or *de facto* rules and in G_n there is an edge from **x** to **y** labeled *r*.

Example



y creates (*rw* to new) **z**

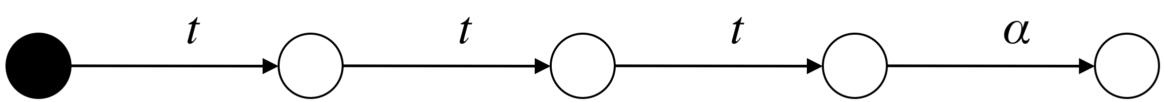
x takes (*r* to **z**) from **y**

y passes to **x** through **z**

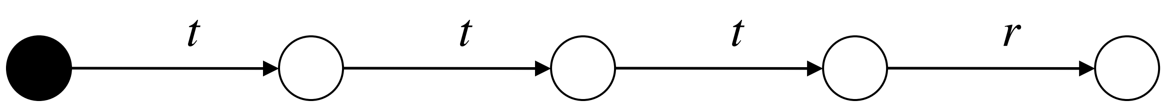
x takes (*w* to **z**) from **y**

y posts to **x** through **z**

Combined Transfers



terminal span

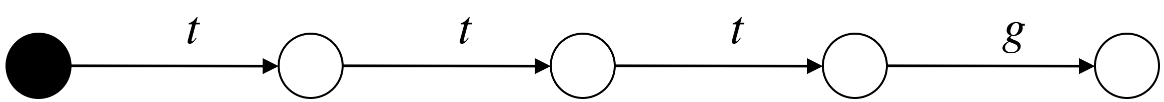


rw-terminal span

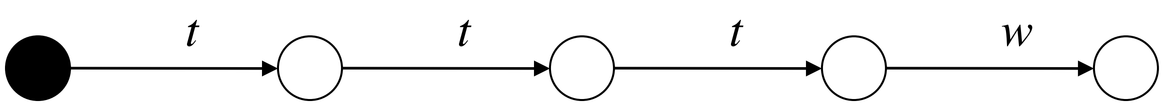
The subject can acquire α rights over the last object

The subject can acquire *r* rights over the last object

Combined Transfers



initial span

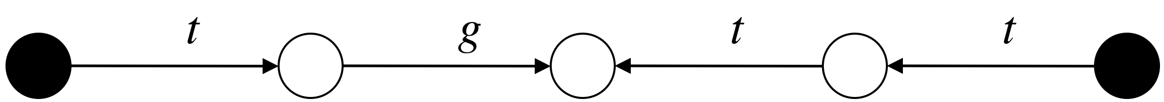


rw-initial span

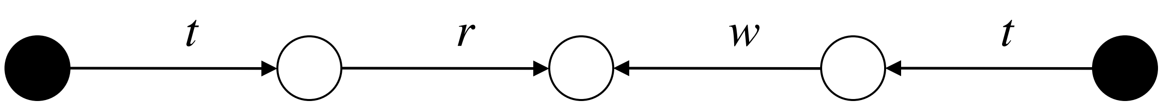
The subject can acquire *g* rights over the last object

The subject can acquire *w* rights over the last object

Combined Transfers



bridge



connection

Just as rights can be transferred over a bridge, information can flow over a connection

Theorem

$can\bullet know(\mathbf{p}, \mathbf{q}, G_0)$ holds if and only if:

(a) $can\bullet share(r, \mathbf{p}, \mathbf{q}, G_0)$ holds, or

(b) there is a sequence of subjects $\mathbf{u_1}, \ldots, \mathbf{u}_n$ such that all of the following are true:

- (i) $\mathbf{p} = \mathbf{u_1}$ or $\mathbf{u_1}$ rw-initially spans to $\mathbf{p}$;
- (ii) $\mathbf{q} = \mathbf{u}_n$ or $\mathbf{u}_n$ rw-terminally spans to $\mathbf{q}$; and
- (iii) for all i, $1 \le i < n$, there is an *rwtg*-path between $\mathbf{u_i}$ and $\mathbf{u_{i+1}}$ with associated word a bridge or connection

Back to Example

can•know($\mathbf{p}$, $\mathbf{q}$, G_0) holds:

•take $n = 2$, $\mathbf{u_1}$ = x, and $\mathbf{u_2}$ = y

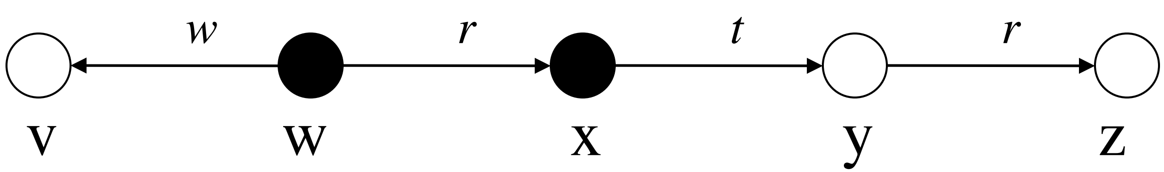
– $\mathbf{p} = \mathbf{u_1}$ or $\mathbf{u_1}$ rw-initially spans to $\mathbf{p}$;

– $\mathbf{q} = \mathbf{u_2}$ or $\mathbf{u_2}$ rw-terminally spans to $\mathbf{q}$; and

– there is an *rwtg*-path between $\mathbf{u_1}$ and $\mathbf{u_2}$ with associated word a bridge or connection

  • $\mathbf{u_1}$, $\mathbf{u_2}$ connected with a *t* edge

Final Example



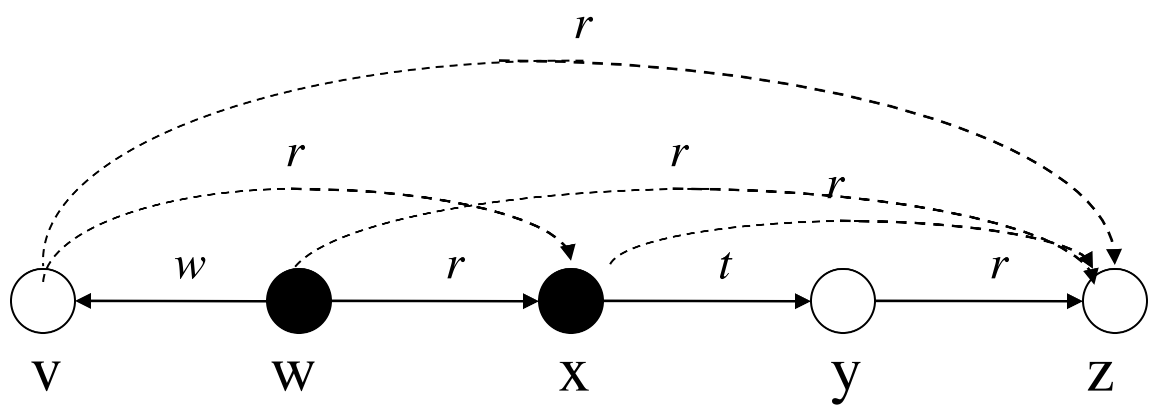
can•share(r, **v**, **z**, G_0) is false

- no initial span between **v** and any subject

can•know(**v**, **z**, G_0) is true

- $\mathbf{u_1} = \mathbf{w}$, $\mathbf{u_2} = \mathbf{x}$
- $\mathbf{u_1}$ *rw*-initially spans to **y**
- $\mathbf{u_2}$ *rw*-terminally spans to **z**
- there is a connection between $\mathbf{u_1}$ and $\mathbf{u_2}$

Final Example Witness



x takes (*r* to **z**) from **y**

w spies on **z** through **x**

w passes from **z** to **v**

x takes (*r* to **z**) from **y**

w passes from **x** to **v**

Key Question

- Characterize class of models for which safety is decidable
  - Existence: Take-Grant Protection Model is a member of such a class
  - Universality: In general, question undecidable, so for some models it is not decidable
- What is the dividing line?

Schematic Protection Model

- Type-based model
  - Protection type: entity label determining how control rights affect the entity
    - Set at creation and cannot be changed
  - Ticket: description of a single right over an entity
    - Entity has sets of tickets (called a *domain*)
    - Ticket is **X**/*r*, where **X** is entity and *r* right
  - Functions determine rights transfer
    - Link: are source, target "connected"?
    - Filter: is transfer of ticket authorized?

Link Predicate

- Idea: $link_i(\mathbf{X}, \mathbf{Y})$ if **X** can assert some control right over **Y**
- Conjunction of disjunction of:
  - $\mathbf{X}/z \in dom(\mathbf{X})$
  - $\mathbf{X}/z \in dom(\mathbf{Y})$
  - $\mathbf{Y}/z \in dom(\mathbf{X})$
  - $\mathbf{Y}/z \in dom(\mathbf{Y})$
  - **true**

Examples

- Take-Grant:

  $link(\mathbf{X}, \mathbf{Y}) = \mathbf{Y}/g \in dom(\mathbf{X}) \vee \mathbf{X}/t \in dom(\mathbf{Y})$

- Broadcast:

  $link(\mathbf{X}, \mathbf{Y}) = \mathbf{X}/b \in dom(\mathbf{X})$

- Pull:

  $link(\mathbf{X}, \mathbf{Y}) = \mathbf{Y}/p \in dom(\mathbf{Y})$

Filter Function

- Range is set of copyable tickets
  - Entity type, right
- Domain is subject pairs
- Copy a ticket $\mathbf{X}/r{:}c$ from $dom(\mathbf{Y})$ to $dom(\mathbf{Z})$
  - $\mathbf{X}/rc \in dom(\mathbf{Y})$
  - $link_i(\mathbf{Y}, \mathbf{Z})$
  - $\tau(\mathbf{Y})/r{:}c \in f_i(\tau(\mathbf{Y}), \tau(\mathbf{Z}))$
- One filter function per link predicate

Example

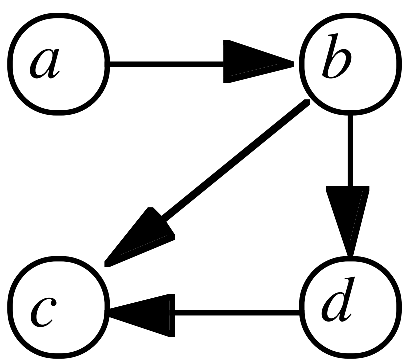
- $f(\tau(\mathbf{Y}), \tau(\mathbf{Z})) = T \times R$
  - Any ticket can be transferred (if other conditions met)
- $f(\tau(\mathbf{Y}), \tau(\mathbf{Z})) = T \times RI$
  - Only tickets with inert rights can be transferred (if other conditions met)
- $f(\tau(\mathbf{Y}), \tau(\mathbf{Z})) = \varnothing$
  - No tickets can be transferred

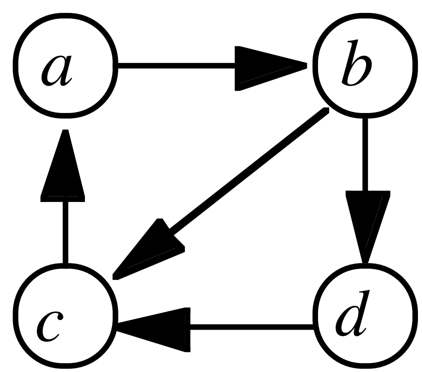
Example

- Take-Grant Protection Model
  - $TS = \{ \text{subjects} \}$, $TO = \{ \text{objects} \}$
  - $RC = \{ tc, gc \}$, $RI = \{ rc, wc \}$
  - $link(\mathbf{p}, \mathbf{q}) = \mathbf{p}/t \in dom(\mathbf{q}) \vee \mathbf{q}/g \in dom(\mathbf{p})$
  - $f(subject, subject) = \{ subject, object \} \times \{ tc, gc, rc, wc \}$

Create Operation

- Must handle type, tickets of new entity
- Relation $cc(a, b)$ [*cc* for *can-create*]
  - Subject of type *a* can create entity of type *b*
- Rule of acyclic creates:





Types

- $cr(a, b)$: tickets created when subject of type a creates entity of type b [*cr* for *create-rule*]
- **B** object: $cr(a, b) \subseteq \{ b/r{:}c \in RI \}$
  - **A** gets **B**/$r{:}c$ iff $b/r{:}c \in cr(a, b)$
- **B** subject: $cr(a, b)$ has two subsets
  - $cr_P(a, b)$ added to **A**, $cr_C(a, b)$ added to **B**
  - **A** gets **B**/$r{:}c$ if $b/r{:}c \in cr_P(a, b)$
  - **B** gets **A**/$r{:}c$ if $a/r{:}c \in cr_C(a, b)$

Non-Distinct Types

$cr(a, a)$: who gets what?

- $self/r{:}c$ are tickets for creator
- $a/r{:}c$ tickets for created

$cr(a, a) = \{ a/r{:}c, self/r{:}c \mid r{:}c \in R \}$

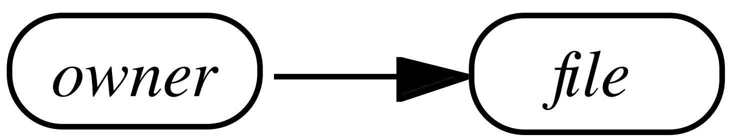
Attenuating Create Rule

$cr(a, b)$ attenuating if:

1. $cr_C(a, b) \subseteq cr_P(a, b)$ and
2. $a/r{:}c \in cr_P(a, b) \Rightarrow self/r{:}c \in cr_P(a, b)$

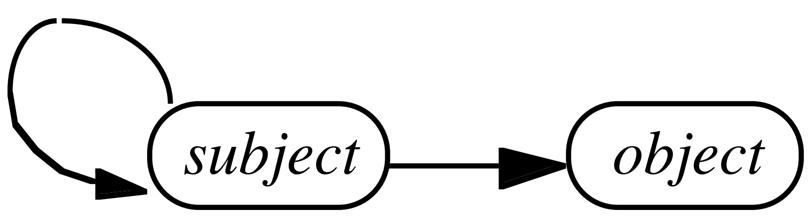
Example: Owner-Based Policy

- Users can create files, creator can give itself any inert rights over file
  - $cc = \{\ (\ user\ ,\ file\ )\ \}$
  - $cr(user, file) = \{\ file/r{:}c \mid r \in RI\ \}$
- Attenuating, as graph is acyclic, loop free



Example: Take-Grant

- Say subjects create subjects (type *s*), objects (type *o*), but get only inert rights over latter
  - $cc = \{ (s, s), (s, o) \}$
  - $cr_C(a, b) = \varnothing$
  - $cr_P(s, s) = \{s/tc, s/gc, s/rc, s/wc\}$
  - $cr_P(s, o) = \{o/rc, o/wc\}$
- Not attenuating, as no *self* tickets provided; *subject* creates *subject*



Safety Analysis

- Goal: identify types of policies with tractable safety analyses
- Approach: derive a state in which additional entries, rights do not affect the analysis; then analyze this state
  - Called a *maximal state*

Definitions

- System begins at initial sate
- Authorized operation causes *legal transition*
- Sequence of legal transitions moves system into final state
  - This sequence is a *history*
  - Final state is *derivable* from history, initial state

More Definitions

- States represented by h
- Set of subjects SUB^h, entities ENT^h
- Link relation in context of state h $link^h$
- Dom relation in context of state h dom^h

$path^h(\mathbf{X},\mathbf{Y})$

- $\mathbf{X}$, $\mathbf{Y}$ connected by one link or a sequence of links
- Formally, either of these hold:
  - for some i, $link_i^h(\mathbf{X}, \mathbf{Y})$; or
  - there is a sequence of subjects $\mathbf{X}_0, \ldots, \mathbf{X}_n$ such that $link_i^h(\mathbf{X}, \mathbf{X}_0)$, $link_i^h(\mathbf{X}_n, \mathbf{Y})$, and for $k = 1, \ldots, n$, $link_i^h(\mathbf{X}_{k-1}, \mathbf{X}_k)$
- If multiple such paths, refer to $path_j^h(\mathbf{X}, \mathbf{Y})$