

# April 7: Safety Question

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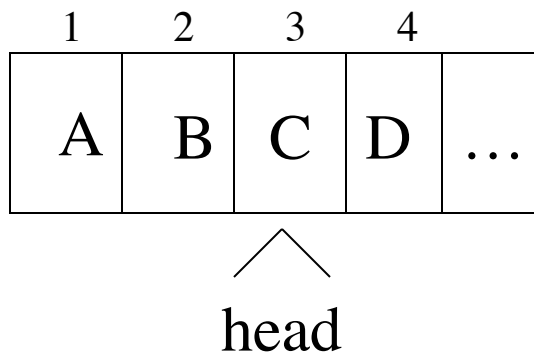
- Protection State Transitions
  - Commands
  - Conditional Commands
- Special Rights
  - Principle of Attenuation of Privilege
- Harrison-Ruzzo-Ullman result
  - Corollaries

# General Case

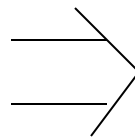
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- Answer: *no*
- Sketch of proof:
  - Reduce halting problem to safety problem
  - Turing Machine review:
    - Infinite tape in one direction
    - States  $K$ , symbols  $M$ ; distinguished blank  $b$
    - Transition function  $\delta(k, m) = (k', m', L)$  means in state  $k$ , symbol  $m$  on tape location replaced by symbol  $m'$ , head moves to left one square, and enters state  $k'$
    - Halting state is  $q_f$ ; TM halts when it enters this state

# Mapping

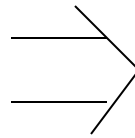
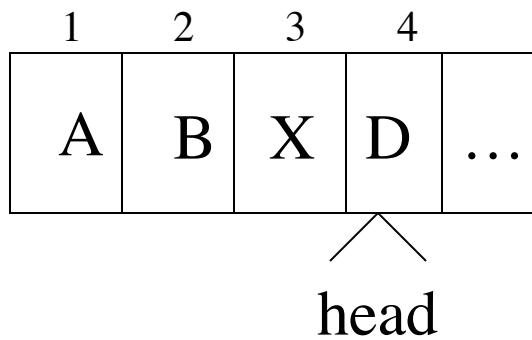


Current state is  $k$



	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			C $k$	<i>own</i>	
$s_4$				D end	

# Mapping



	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			X	<i>own</i>	
$s_4$				D $k_1$ end	

After  $\delta(k, C) = (k_1, X, R)$   
 where  $k$  is the current  
 state and  $k_1$  the next state

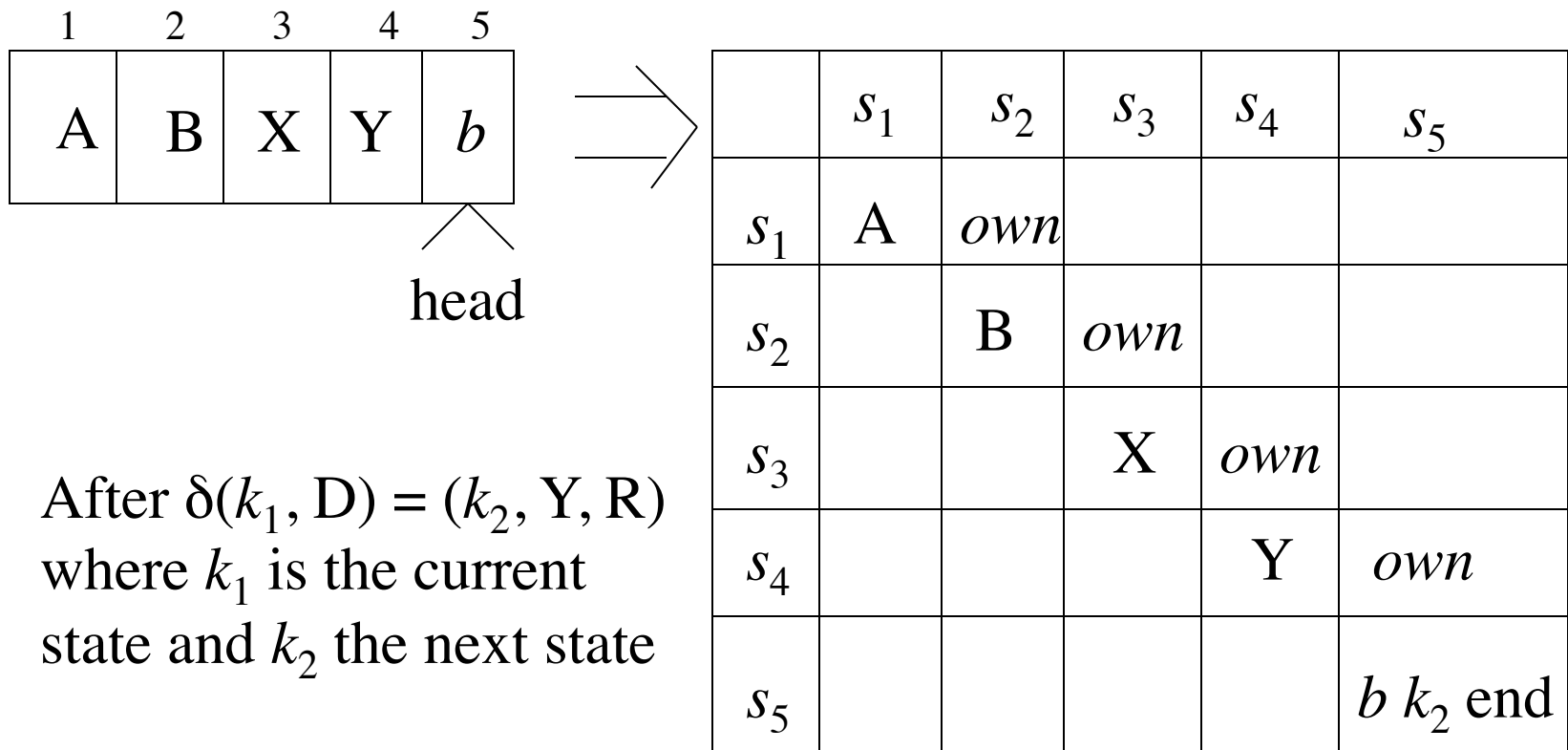
# Command Mapping

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$\delta(k, C) = (k_1, X, R)$  at intermediate becomes

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command  $c_{k,C}(s_3, s_4)$   
if own in  $A[s_3, s_4]$  and  $k$  in  $A[s_3, s_3]$   
    and  $C$  in  $A[s_3, s_3]$   
then  
    delete  $k$  from  $A[s_3, s_3]$ ;  
    delete  $C$  from  $A[s_3, s_3]$ ;  
    enter  $X$  into  $A[s_3, s_3]$ ;  
    enter  $k_1$  into  $A[s_4, s_4]$ ;  
end
```

# Mapping



After  $\delta(k_1, D) = (k_2, Y, R)$   
 where  $k_1$  is the current  
 state and  $k_2$  the next state

# Command Mapping

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$\delta(k_1, D) = (k_2, Y, R)$  at end becomes

```
command crightmostk,c(s4, s5)  
if end in A[s4, s4] and k1 in A[s4, s4]  
    and D in A[s4, s4]  
then  
    delete end from A[s4, s4];  
    delete k1 from A[s4, s4];  
    delete D from A[s4, s4];  
    enter Y into A[s4, s4];  
    create subject s5;  
    enter own into A[s4, s5];  
    enter end into A[s5, s5];  
    enter k2 into A[s5, s5];  
end
```

# Rest of Proof

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- Protection system exactly simulates a TM
  - Exactly 1 *end* right in ACM
  - 1 right in entries corresponds to state
  - Thus, at most 1 applicable command
- If TM enters state  $q_f$ , then right has leaked
- If safety question decidable, then represent TM as above and determine if  $q_f$  leaks
  - Implies halting problem decidable
- Conclusion: safety question undecidable



# Other Results

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- Set of unsafe systems is recursively enumerable
- Delete **create** primitive; then safety question is complete in **P-SPACE**
- Delete **destroy**, **delete** primitives; then safety question is undecidable
  - Systems are monotonic
- Safety question for biconditional protection systems is decidable
- Safety question for monoconditional, monotonic protection systems is decidable
- Safety question for monoconditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.

# Take-Grant Protection Model

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- A specific (not generic) system
  - Set of rules for state transitions
- Safety decidable, and in time linear with the size of the system
- Goal: find conditions under which rights can be transferred from one entity to another in the system

# System

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- objects (files, ...)
- subjects (users, processes, ...)
- ⊗ don't care (either a subject or an object)

$G \vdash_x G'$       apply a rewriting rule  $x$  (witness) to  $G$  to get  $G'$

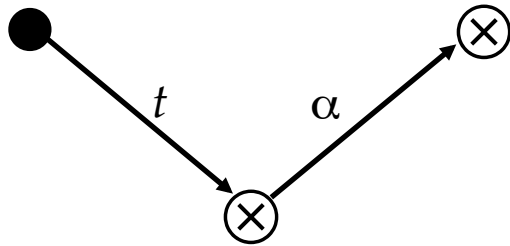
$G \vdash^* G'$       apply a sequence of rewriting rules (witness) to  $G$  to get  $G'$

$R = \{ t, g, r, w, \dots \}$     set of rights

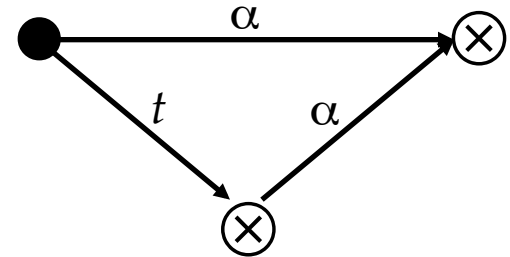
# Rules

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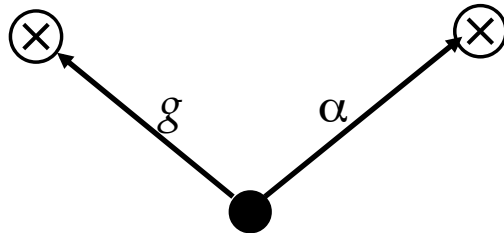
take



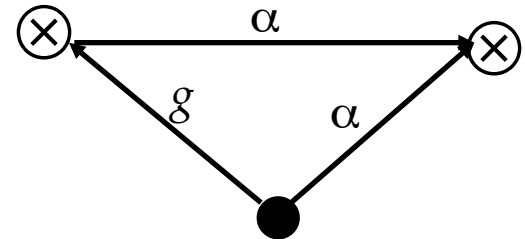
$\vdash$



grant



$\vdash$



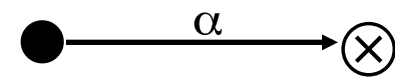
# More Rules

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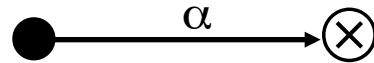
create



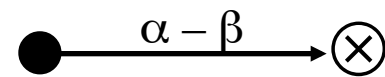
$\vdash$



remove



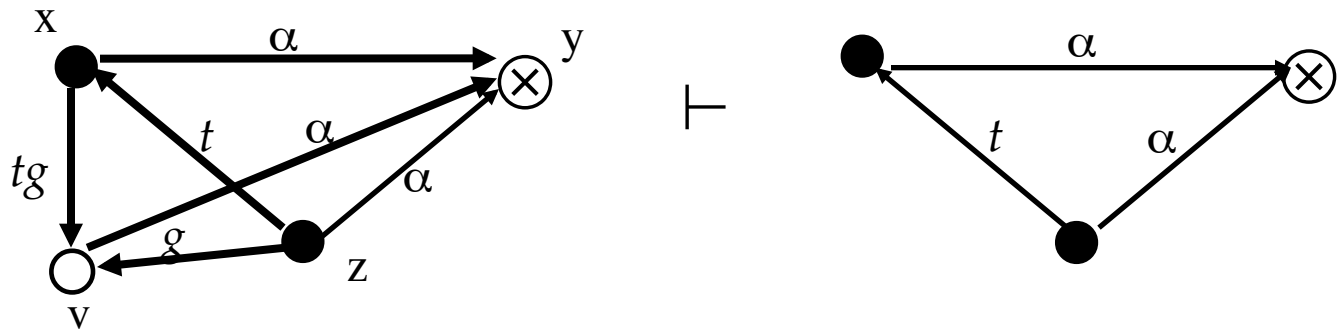
$\vdash$



These four rules are called the *de jure* rules

# Symmetry

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1.  $x$  creates ( $tg$  to new)  $v$
2.  $z$  takes ( $g$  to  $v$ ) from  $x$
3.  $z$  grants ( $\alpha$  to  $y$ ) to  $v$
4.  $x$  takes ( $\alpha$  to  $y$ ) from  $v$

Similar result for grant

# Islands

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- $tg$ -path: path of distinct vertices connected by edges labeled  $t$  or  $g$ 
  - Call them “ $tg$ -connected”
- island: maximal  $tg$ -connected subject-only subgraph
  - Any right one vertex has can be shared with any other vertex

# Initial, Terminal Spans

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- *initial span* from  $\mathbf{x}$  to  $\mathbf{y}$ 
  - $\mathbf{x}$  subject
  - $tg$ -path between  $\mathbf{x}$ ,  $\mathbf{y}$  with word in  $\{ \vec{t^*g} \} \cup \{ \mathbf{v} \}$
  - Means  $\mathbf{x}$  can give rights it has to  $\mathbf{y}$
- *terminal span* from  $\mathbf{x}$  to  $\mathbf{y}$ 
  - $\mathbf{x}$  subject
  - $tg$ -path between  $\mathbf{x}$ ,  $\mathbf{y}$  with word in  $\{ \vec{t^*} \} \cup \{ \mathbf{v} \}$
  - Means  $\mathbf{x}$  can acquire any rights  $\mathbf{y}$  has



# Bridges

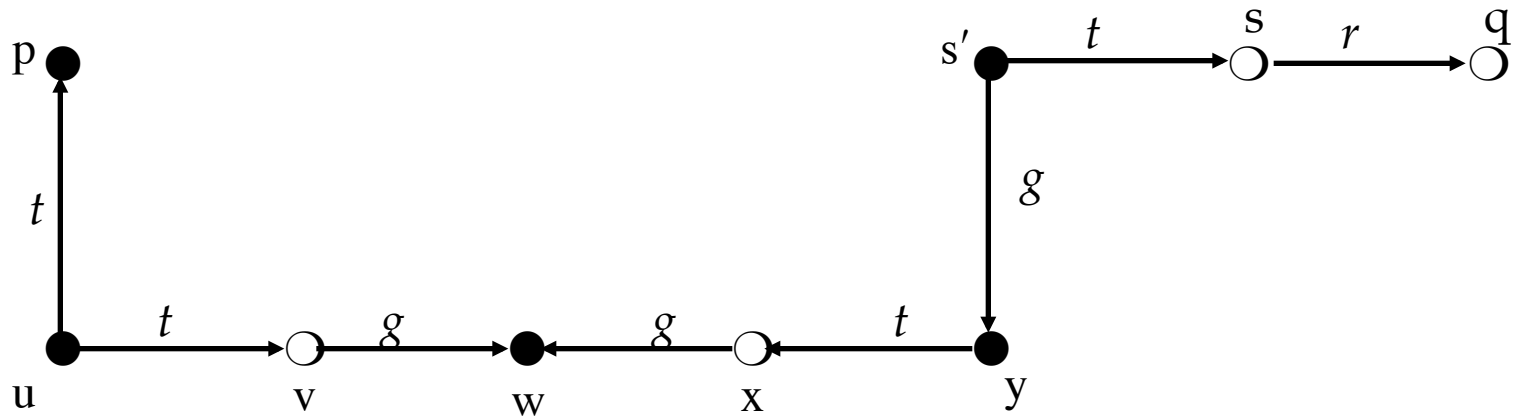
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- bridge:  $tg$ -path between subjects  $\mathbf{x}$ ,  $\mathbf{y}$ , with associated word in

$$\{ \vec{t}^*, \bar{t}^*, \vec{t}^* \overleftarrow{g} \bar{t}^*, \vec{t}^* \overrightarrow{g} \bar{t}^* \}$$

- rights can be transferred between the two endpoints
- *not* an island as intermediate vertices are objects

# Example



- islands  $\{ p, u \}$   $\{ w \}$   $\{ y, s' \}$
- bridges  $u, v, w; w, x, y$
- initial span  $p$  (associated word  $v$ )
- terminal span  $s'$ s (associated word  $\vec{t}$ )

# can•share Predicate

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Definition:

- $can\bullet share(r, \mathbf{x}, \mathbf{y}, G_0)$  if, and only if, there is a sequence of protection graphs  $G_0, \dots, G_n$  such that  $G_0 \vdash^* G_n$  using only *de jure* rules and in  $G_n$  there is an edge from  $\mathbf{x}$  to  $\mathbf{y}$  labeled  $r$ .

# *can•share* Theorem

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- *can•share*( $r, \mathbf{x}, \mathbf{y}, G_0$ ) if, and only if, there is an edge from  $\mathbf{x}$  to  $\mathbf{y}$  labeled  $r$  in  $G_0$ , or the following hold simultaneously:
  - There is an  $\mathbf{s}$  in  $G_0$  with an  $\mathbf{s}$ -to- $\mathbf{y}$  edge labeled  $r$
  - There is a subject  $\mathbf{x}' = \mathbf{x}$  or initially spans to  $\mathbf{x}$
  - There is a subject  $\mathbf{s}' = \mathbf{s}$  or terminally spans to  $\mathbf{s}$
  - There are islands  $I_1, \dots, I_k$  connected by bridges, and  $\mathbf{x}'$  in  $I_1$  and  $\mathbf{s}'$  in  $I_k$

# Outline of Proof

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- $s$  has  $r$  rights over  $y$
- $s'$  acquires  $r$  rights over  $y$  from  $s$ 
  - Definition of terminal span
- $x'$  acquires  $r$  rights over  $y$  from  $s'$ 
  - Repeated application of sharing among vertices in islands, passing rights along bridges
- $x'$  gives  $r$  rights over  $y$  to  $x$ 
  - Definition of initial span

# Example Interpretation

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- ACM is generic
  - Can be applied in any situation
- Take-Grant has specific rules, rights
  - Can be applied in situations matching rules, rights
- Question: what states can evolve from a system that is modeled using the Take-Grant Model?

# Take-Grant Generated Systems

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- Theorem:  $G_0$  protection graph with 1 vertex, no edges;  $R$  set of rights. Then  $G_0 \vdash^* G$  iff:
  - $G$  finite directed graph consisting of subjects, objects, edges
  - Edges labeled from nonempty subsets of  $R$
  - At least one vertex in  $G$  has no incoming edges

# Outline of Proof

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$\Rightarrow$ : By construction;  $G$  final graph in theorem

- Let  $\mathbf{x}_1, \dots, \mathbf{x}_n$  be subjects in  $G$
- Let  $\mathbf{x}_1$  have no incoming edges

• Now construct  $G'$  as follows:

1. Do “ $\mathbf{x}_1$  creates  $(\alpha \cup \{g\})$  to new subject  $\mathbf{x}_i$ ”
2. For all  $(\mathbf{x}_i, \mathbf{x}_j)$  where  $\mathbf{x}_i$  has a rights over  $\mathbf{x}_j$ , do “ $\mathbf{x}_1$  grants  $(\alpha$  to  $\mathbf{x}_j)$  to  $\mathbf{x}_i$ ”
3. Let  $\beta$  be rights  $\mathbf{x}_i$  has over  $\mathbf{x}_j$  in  $G$ . Do “ $\mathbf{x}_1$  removes  $((\alpha \cup \{g\} - \beta)$  to)  $\mathbf{x}_j$ ”

• Now  $G'$  is desired  $G$



# Outline of Proof

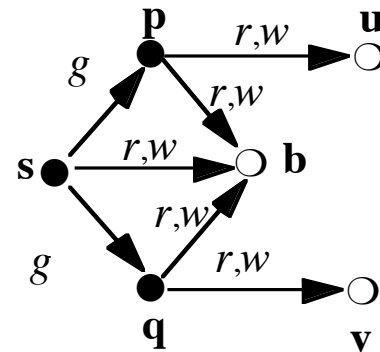
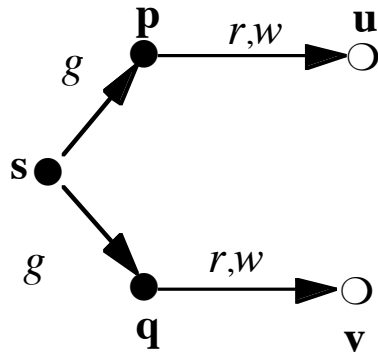
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$\Leftarrow$ : Let  $\mathbf{v}$  be initial subject, and  $G_0 \vdash^* G$

- Inspection of rules gives:
  - $G$  is finite
  - $G$  is a directed graph
  - Subjects and objects only
  - All edges labeled with nonempty subsets of  $R$
- Limits of rules:
  - None allow vertices to be deleted so  $\mathbf{v}$  in  $G$
  - None add incoming edges to vertices without incoming edges, so  $\mathbf{v}$  has no incoming edges

# Example: Shared Buffer

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- Goal: **p**, **q** to communicate through shared buffer **b** controlled by trusted entity **s**
  1. **s** creates (  $\{r, w\}$  to new object) **b**
  2. **s** grants (  $\{r, w\}$  to **b**) to **p**
  3. **s** grants (  $\{r, w\}$  to **b**) to **q**

# Key Question

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- Characterize class of models for which safety is decidable
  - Existence: Take-Grant Protection Model is a member of such a class
  - Universality: In general, question undecidable, so for some models it is not decidable
- What is the dividing line?

# Schematic Protection Model

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- Type-based model
  - Protection type: entity label determining how control rights affect the entity
    - Set at creation and cannot be changed
  - Ticket: description of a single right over an entity
    - Entity has sets of tickets (called a *domain*)
    - Ticket is  $\mathbf{X}/r$ , where  $\mathbf{X}$  is entity and  $r$  right
  - Functions determine rights transfer
    - Link: are source, target “connected”?
    - Filter: is transfer of ticket authorized?

# Link Predicate

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- Idea:  $link_i(\mathbf{X}, \mathbf{Y})$  if  $\mathbf{X}$  can assert some control right over  $\mathbf{Y}$
- Conjunction of disjunction of:
  - $\mathbf{X}/z \in dom(\mathbf{X})$
  - $\mathbf{X}/z \in dom(\mathbf{Y})$
  - $\mathbf{Y}/z \in dom(\mathbf{X})$
  - $\mathbf{Y}/z \in dom(\mathbf{Y})$
  - **true**

# Examples

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- Take-Grant:

$$\mathit{link}(\mathbf{X}, \mathbf{Y}) = \mathbf{Y}/g \in \mathit{dom}(\mathbf{X}) \vee \mathbf{X}/t \in \mathit{dom}(\mathbf{Y})$$

- Broadcast:

$$\mathit{link}(\mathbf{X}, \mathbf{Y}) = \mathbf{X}/b \in \mathit{dom}(\mathbf{X})$$

- Pull:

$$\mathit{link}(\mathbf{X}, \mathbf{Y}) = \mathbf{Y}/p \in \mathit{dom}(\mathbf{Y})$$

# Filter Function

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- Range is set of copyable tickets
  - Entity type, right
- Domain is subject pairs
- Copy a ticket  $\mathbf{X}/r:c$  from  $dom(\mathbf{Y})$  to  $dom(\mathbf{Z})$ 
  - $\mathbf{X}/rc \in dom(\mathbf{Y})$
  - $link_i(\mathbf{Y}, \mathbf{Z})$
  - $\tau(\mathbf{Y})/r:c \in f_i(\tau(\mathbf{Y}), \tau(\mathbf{Z}))$
- One filter function per link function

# Example

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- $f(\tau(\mathbf{Y}), \tau(\mathbf{Z})) = T \times R$ 
  - Any ticket can be transferred (if other conditions met)
- $f(\tau(\mathbf{Y}), \tau(\mathbf{Z})) = T \times RI$ 
  - Only tickets with inert rights can be transferred (if other conditions met)
- $f(\tau(\mathbf{Y}), \tau(\mathbf{Z})) = \emptyset$ 
  - No tickets can be transferred



# Example

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- Take-Grant Protection Model
  - $TS = \{ \text{subjects} \}, TO = \{ \text{objects} \}$
  - $RC = \{ tc, gc \}, RI = \{ rc, wc \}$
  - $link(\mathbf{p}, \mathbf{q}) = \mathbf{p}/t \in dom(\mathbf{q}) \vee \mathbf{q}/g \in dom(\mathbf{p})$
  - $f(\text{subject}, \text{subject}) = \{ \text{subject}, \text{object} \} \times \{ tc, gc, rc, wc \}$