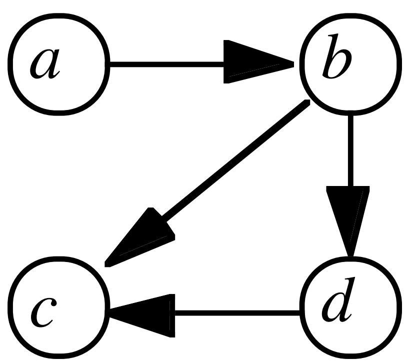
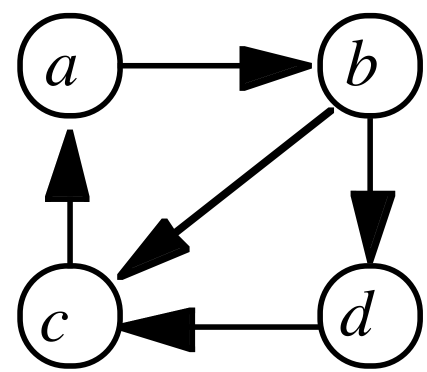
April 10: Expressiveness

- SPM and safety

Create Operation

- Must handle type, tickets of new entity
- Relation $cc(a, b)$ [*cc* for *can-create*]
  - Subject of type *a* can create entity of type *b*
- Rule of acyclic creates:





Types

- $cr(a, b)$: tickets created when subject of type a creates entity of type b [*cr* for *create-rule*]
- **B** object: $cr(a, b) \subseteq \{ b/r{:}c \in RI \}$
  - **A** gets **B**/$r{:}c$ iff $b/r{:}c \in cr(a, b)$
- **B** subject: $cr(a, b)$ has two subsets
  - $cr_P(a, b)$ added to **A**, $cr_C(a, b)$ added to **B**
  - **A** gets **B**/$r{:}c$ if $b/r{:}c \in cr_P(a, b)$
  - **B** gets **A**/$r{:}c$ if $a/r{:}c \in cr_C(a, b)$

Non-Distinct Types

$cr(a, a)$: who gets what?

- $self/r{:}c$ are tickets for creator
- $a/r{:}c$ tickets for created

$cr(a, a) = \{ a/r{:}c, self/r{:}c \mid r{:}c \in R\}$

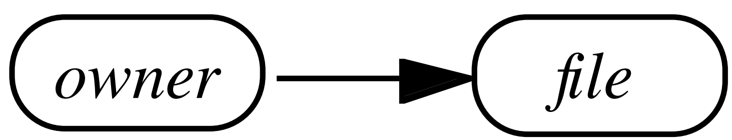
Attenuating Create Rule

$cr(a, b)$ attenuating if:

1. $cr_C(a, b) \subseteq cr_P(a, b)$ and
2. $a/r{:}c \in cr_P(a, b) \Rightarrow self/r{:}c \in cr_P(a, b)$

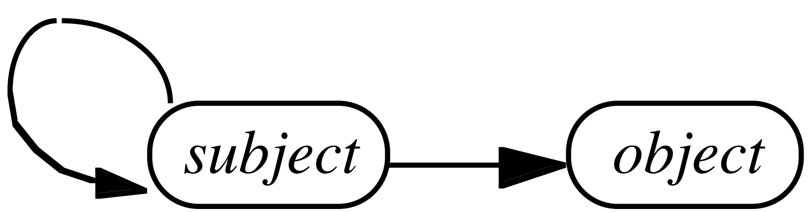
Example: Owner-Based Policy

- Users can create files, creator can give itself any inert rights over file
  - $cc = \{ (user, file) \}$
  - $cr(user, file) = \{ file/r{:}c \mid r \in RI \}$
- Attenuating, as graph is acyclic, loop free



Example: Take-Grant

- Say subjects create subjects (type *s*), objects (type *o*), but get only inert rights over latter
  - $cc = \{ (s, s), (s, o) \}$
  - $cr_C(a, b) = \varnothing$
  - $cr_P(s, s) = \{s/tc, s/gc, s/rc, s/wc \}$
  - $cr_P(s, o) = \{s/rc, s/wc \}$
- Not attenuating, as no *self* tickets provided; *subject* creates *subject*



Safety Analysis

- Goal: identify types of policies with tractable safety analyses
- Approach: derive a state in which additional entries, rights do not affect the analysis; then analyze this state
  - Called a *maximal state*

Definitions

- System begins at initial state
- Authorized operation causes *legal transition*
- Sequence of legal transitions moves system into final state
  - This sequence is a *history*
  - Final state is *derivable* from history, initial state

More Definitions

- States represented by h
- Set of subjects SUB^h, entities ENT^h
- Link relation in context of state h is $link^h$
- Dom relation in context of state h is dom^h

$path^h(\mathbf{X},\mathbf{Y})$

- **X**, **Y** connected by one link or a sequence of links
- Formally, either of these hold:
  - for some i, $link_i^h(\mathbf{X}, \mathbf{Y})$; or
  - there is a sequence of subjects $\mathbf{X}_0, \ldots, \mathbf{X}_n$ such that $link_i^h(\mathbf{X}, \mathbf{X}_0)$, $link_i^h(\mathbf{X}_n, \mathbf{Y})$, and for $k = 1, \ldots, n$, $link_i^h(\mathbf{X}_{k-1}, \mathbf{X}_k)$
- If multiple such paths, refer to $path_j^h(\mathbf{X}, \mathbf{Y})$

Capacity $cap(path^h(\mathbf{X},\mathbf{Y}))$

- Set of tickets that can flow over $path^h(\mathbf{X},\mathbf{Y})$
  - If $link_i^h(\mathbf{X},\mathbf{Y})$: set of tickets that can be copied over the link (i.e., $f_i(\tau(\mathbf{X}), \tau(\mathbf{Y}))$)
  - Otherwise, set of tickets that can be copied over *all* links in the sequence of links making up the $path^h(\mathbf{X},\mathbf{Y})$
- Note: all tickets (except those for the final link) *must* be copyable

Flow Function

- Idea: capture flow of tickets around a given state of the system
- Let there be *m* $path^h$s between subjects **X** and **Y** in state *h*. Then *flow function*

$$flow^h: SUB^h \times SUB^h \rightarrow 2^{T \times R}$$

is:

$$flow^h(\mathbf{X},\mathbf{Y}) = \bigcup_{i=1,\ldots,m} cap(path_i^h(\mathbf{X},\mathbf{Y}))$$

Properties of Maximal State

- Maximizes flow between all pairs of subjects
  - State is called *
  - Ticket in *flow**(**X**,**Y**) means there exists a sequence of operations that can copy the ticket from **X** to **Y**
- Questions
  - Is maximal state unique?
  - Does every system have one?

Formal Definition

- Definition: $g \leq_0 h$ holds iff for all $\mathbf{X}, \mathbf{Y} \in SUB^0$, $\mathit{flow}^g(\mathbf{X},\mathbf{Y}) \subseteq \mathit{flow}^h(\mathbf{X},\mathbf{Y})$.
  - Note: if $g \leq_0 h$ and $h \leq_0 g$, then g, h equivalent
  - Defines set of equivalence classes on set of derivable states
- Definition: for a given system, state m is maximal iff $h \leq_0 m$ for every derivable state h
- Intuition: flow function contains all tickets that can be transferred from one subject to another
  - All maximal states in same equivalence class

Maximal States

- Lemma. Given arbitrary finite set of states H, there exists a derivable state m such that for all $h \in H$, $h \leq_0 m$
- Outline of proof: induction
  - Basis: $H = \varnothing$; trivially true
  - Step: $|H'| = n + 1$, where $H' = G \cup \{h\}$. By IH, there is a $g \in G$ such that $x \leq_0 g$ for all $x \in G$.

Outline of Proof

- M interleaving histories of g, h which:
  - Preserves relative order of transitions in g, h
  - Omits second create operation if duplicated
- M ends up at state m
- If $path^g(\mathbf{X},\mathbf{Y})$ for $\mathbf{X}, \mathbf{Y} \in SUB^g$, $path^m(\mathbf{X},\mathbf{Y})$
  - So $g \leq_0 m$
- If $path^h(\mathbf{X},\mathbf{Y})$ for $\mathbf{X}, \mathbf{Y} \in SUB^h$, $path^m(\mathbf{X},\mathbf{Y})$
  - So $h \leq_0 m$
- Hence m maximal state in H'

Answer to Second Question

- Theorem: every system has a maximal state *
- Outline of proof: *K* is set of derivable states containing exactly one state from each equivalence class of derivable states
  - Consider **X**, **Y** in SUB^0. Flow function's range is $2^{T \times R}$, so can take at most $2^{|T \times R|}$ values. As there are $|SUB^0|^2$ pairs of subjects in SUB^0, at most $2^{|T \times R|} |SUB^0|^2$ distinct equivalence classes; so *K* is finite
- Result follows from lemma

Safety Question

- In this model:

  Is it possible to have a derivable state with **X**/*r*:*c* in *dom*(**A**), or does there exist a subject **B** with ticket **X**/*rc* in the initial state or which can demand **X**/*rc* and τ(**X**)/*r*:*c* in *flow**(**B**,**A**)?
- To answer: construct maximal state and test
  - Consider acyclic attenuating schemes; how do we construct maximal state?

Intuition

- Consider state h.
- State u corresponds to h but with minimal number of new entities created such that maximal state m can be derived with no create operations
  - So if in history from h to m, subject **X** creates two entities of type a, in u only one would be created; surrogate for both
- m can be derived from u in polynomial time, so if u can be created by adding a finite number of subjects to h, safety question decidable.

Fully Unfolded State

- State u derived from state 0 as follows:
  - delete all loops in cc; new relation cc'
  - mark all subjects as folded
  - while any $\mathbf{X} \in SUB^0$ is folded
    - mark it unfolded
    - if **X** can create entity **Y** of type y, it does so (call this the y-surrogate of **X**); if entity $\mathbf{Y} \in SUB^g$, mark it folded
  - if any subject in state h can create an entity of its own type, do so
- Now in state u

Termination

- First loop terminates as SUB^0 finite
- Second loop terminates:
  - Each subject in SUB^0 can create at most $| TS |$ children, and $| TS |$ is finite
  - Each folded subject in $| SUB^i |$ can create at most $| TS | - i$ children
  - When $i = | TS |$, subject cannot create more children; thus, folded is finite
  - Each loop removes one element
- Third loop terminates as SUB^h is finite

Surrogate

- Intuition: surrogate collapses multiple subjects of same type into single subject that acts for all of them
- Definition: given initial state 0, for every derivable state h define *surrogate function* $\sigma: ENT^h \rightarrow ENT^h$ by:
  – if $\mathbf{X}$ in ENT^0, then $\sigma(\mathbf{X}) = \mathbf{X}$
  – if $\mathbf{Y}$ creates $\mathbf{X}$ and $\tau(\mathbf{Y}) = \tau(\mathbf{X})$, then $\sigma(\mathbf{X}) = \sigma(\mathbf{Y})$
  – if $\mathbf{Y}$ creates $\mathbf{X}$ and $\tau(\mathbf{Y}) \neq \tau(\mathbf{X})$, then $\sigma(\mathbf{X}) = \tau(\mathbf{Y})$-surrogate of $\sigma(\mathbf{Y})$

Implications

- $\tau(\sigma(\mathbf{X})) = \tau(\mathbf{X})$
- If $\tau(\mathbf{X}) = \tau(\mathbf{Y})$, then $\sigma(\mathbf{X}) = \sigma(\mathbf{Y})$
- If $\tau(\mathbf{X}) \neq \tau(\mathbf{Y})$, then
  - $\sigma(\mathbf{X})$ creates $\sigma(\mathbf{Y})$ in the construction of u
  - $\sigma(\mathbf{X})$ creates entities $\mathbf{X}'$ of type $\tau(\mathbf{X}') = \tau(\sigma(\mathbf{X}))$
- From these, for a system with an acyclic attenuating scheme, if **X** creates **Y**, then tickets that would be introduced by pretending that $\sigma(\mathbf{X})$ creates $\sigma(\mathbf{Y})$ are in $dom^u(\sigma(\mathbf{X}))$ and $dom^u(\sigma(\mathbf{Y}))$

Deriving Maximal State

- Idea
  - Reorder operations so that all creates come first and replace history with equivalent one using surrogates
  - Show maximal state of new history is also that of original history
  - Show maximal state can be derived from initial state

Reordering

- *H* legal history deriving state *h* from state 0
- Order operations: first create, then demand, then copy operations
- Build new history *G* from *H* as follows:
  - Delete all creates
  - "**X** demands **Y**/*r*:*c*" becomes "σ(**X**) demands σ(**Y**)/*r*:*c*"
  - "**Y** copies **X** /*r*:*c* from **Y**" becomes "σ(**Y**) copies σ(**X**)/*r*:*c* from σ(**Y**)"

Tickets in Parallel

- Theorem
  - All transitions in G legal; if $\mathbf{X}/r{:}c \in dom^h(Y)$, then $\sigma(\mathbf{X})/r{:}c \in dom^h(\sigma(\mathbf{Y}))$
- Outline of proof: induct on number of copy operations in H

Basis

- H has create, demand only; so G has demand only. s preserves type, so by construction every demand operation in G legal.
- 3 ways for $\mathbf{X}/r{:}c$ to be in $dom^h(\mathbf{Y})$:
  - $\mathbf{X}/r{:}c \in dom^0(\mathbf{Y})$ means $\mathbf{X}, \mathbf{Y} \in ENT^0$, so trivially $\sigma(\mathbf{X})/r{:}c \in dom^g(\sigma(\mathbf{Y}))$ holds
  - A create added $\mathbf{X}/r{:}c \in dom^h(\mathbf{Y})$: previous lemma says $\sigma(\mathbf{X})/r{:}c \in dom^g(\sigma(\mathbf{Y}))$ holds
  - A demand added $\mathbf{X}/r{:}c \in dom^h(\mathbf{Y})$: corresponding demand operation in G gives $\sigma(\mathbf{X})/r{:}c \in dom^g(\sigma(\mathbf{Y}))$

Hypothesis

- Claim holds for all histories with k copy operations
- History H has k+1 copy operations
  - H' initial sequence of H composed of k copy operations
  - h' state derived from H'

Step

- G' sequence of modified operations corresponding to H'; g' derived state
  - G' legal history by hypothesis
- Final operation is "Z copied X/r:c from Y"
  - So h, h' differ by at most $\mathbf{X}/r{:}c \in dom^h(\text{Z})$
  - Construction of G means final operation is $\sigma(\mathbf{X})/r{:}c \in dom^g(\sigma(\mathbf{Y}))$
- Proves second part of claim

Step

- H' legal, so for H to be legal, we have:
  1. $\mathbf{X}/rc \in dom^{h'}(\mathbf{Y})$
  2. $link_i^{h'}(\mathbf{Y}, \mathbf{Z})$
  3. $\tau(\mathbf{X}/r{:}c) \in f_i(\tau(\mathbf{Y}), \tau(\mathbf{Z}))$
- By IH, 1, 2, as $\mathbf{X}$/r:c $\in dom^{h'}(\mathbf{Y})$,

$$\sigma(\mathbf{X})/r{:}c \in dom^{g'}(\sigma(\mathbf{Y})) \text{ and } link_i^{g'}(\sigma(\mathbf{Y}), \sigma(\mathbf{Z}))$$

- As σ preserves type, IH and 3 imply

$$\tau(\sigma(\mathbf{X})/r{:}c) \in f_i(\tau((\sigma(\mathbf{Y})), \tau(\sigma(\mathbf{Z})))$$

- IH says G' legal, so G is legal

Corollary

- If $link_i^h(\mathbf{X}, \mathbf{Y})$, then $link_i^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y}))$

Main Theorem

- System has acyclic attenuating scheme
- For every history *H* deriving state *h* from initial state, there is a history *G* without create operations that derives *g* from the fully unfolded state *u* such that

$$(\forall \mathbf{X},\mathbf{Y} \in SUB^h)[flow^h(\mathbf{X}, \mathbf{Y}) \subseteq flow^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y}))]$$

- Meaning: any history derived from an initial statecan be simulated by corresponding history applied to the fully unfolded state derived from the initial state

Proof

- Outline of proof: show that every $path^h(\mathbf{X},\mathbf{Y})$ has corresponding $path^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y}))$ such that $cap(path^h(\mathbf{X},\mathbf{Y})) = cap(path^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y})))$
  - Then corresponding sets of tickets flow through systems derived from *H* and *G*
  - As initial states correspond, so do those systems
- Proof by induction on number of links

Basis and Hypothesis

- Length of $path^h(\mathbf{X}, \mathbf{Y}) = 1$. By definition of $path^h$, $link_i^h(\mathbf{X}, \mathbf{Y})$, hence $link_i^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y}))$. As σ preserves type, this means

$$cap(path^h(\mathbf{X}, \mathbf{Y})) = cap(path^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y})))$$

- Now assume this is true when $path^h(\mathbf{X}, \mathbf{Y})$ has length k

Step

- Let $\mathit{path}^h(\mathbf{X}, \mathbf{Y})$ have length $k+1$. Then there is a $\mathbf{Z}$ such that $\mathit{path}^h(\mathbf{X}, \mathbf{Z})$ has length k and $\mathit{link}_j^h(\mathbf{Z}, \mathbf{Y})$.
- By IH, there is a $\mathit{path}^g(\sigma(\mathbf{X}), \sigma(\mathbf{Z}))$ with same capacity as $\mathit{path}^h(\mathbf{X}, \mathbf{Z})$
- By corollary, $\mathit{link}_j^g(\sigma(\mathbf{Z}), \sigma(\mathbf{Y}))$
- As σ preserves type, there is $\mathit{path}^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y}))$ with

$$\mathit{cap}(\mathit{path}^h(\mathbf{X}, \mathbf{Y})) = \mathit{cap}(\mathit{path}^g(\sigma(\mathbf{X}), \sigma(\mathbf{Y})))$$

Implication

- Let maximal state corresponding to v be $\#u$
  - Deriving history has no creates
  - By theorem,

$$(\forall \mathbf{X},\mathbf{Y} \in SUB^h)[flow^h(\mathbf{X}, \mathbf{Y}) \subseteq flow^{\#u}(\sigma(\mathbf{X}), \sigma(\mathbf{Y}))]$$

  - If $\mathbf{X} \in SUB^0$, $\sigma(\mathbf{X}) = \mathbf{X}$, so:

$$(\forall \mathbf{X},\mathbf{Y} \in SUB^0)[flow^h(\mathbf{X}, \mathbf{Y}) \subseteq flow^{\#u}(\mathbf{X}, \mathbf{Y})]$$

- So $\#u$ is maximal state for system with acyclic attenuating scheme
  - $\#u$ derivable from u in time polynomial to $|SUB^u|$
  - Worst case computation for $flow^{\#u}$ is exponential in $|TS|$

Safety Result

- If the scheme is acyclic and attenuating, the safety question is decidable