ECS 235B Module 6
HRU Result

What Is “Secure”?

- Adding a generic right *r* where there was not one is “leaking”
  - In what follows, a right leaks if it was not present *initially*
  - Alternately: not present *in the previous state* (not discussed here)
- If a system *S*, beginning in initial state s_0, cannot leak right *r*, it is *safe with respect to the right r*
  - Otherwise it is called *unsafe with respect to the right r*

Safety Question

- Is there an algorithm for determining whether a protection system S with initial state s_0 is safe with respect to a generic right r?
  - Here, “safe” = “secure” for an abstract model

Mono-Operational Commands

- Answer: *yes*
- Sketch of proof:

  Consider minimal sequence of commands c_1, …, c_k to leak the right.

  - Can omit **delete**, **destroy** (with some rewriting)
  - Can merge all **create**s into one

  Worst case: insert every right into every entry; with s subjects and o objects initially, and n rights; upper bound is $k \leq n(s+1)(o+1)+1$

General Case

- Answer: *no*
- Sketch of proof:

  Reduce halting problem to safety problem

  Turing Machine review:

  - Infinite tape in one direction
  - States K, symbols M; distinguished blank b
  - Transition function $\delta(k, m) = (k', m', \mathrm{L})$ means in state k, symbol m on tape location replaced by symbol m', head moves to left one square, and enters state k'
  - Halting state is q_f; TM halts when it enters this state

Mapping

1	2	3	4	
A	B	C	D	…

head

Current state is *k*

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C *k*	*own*	
s_4				D *end*	

Mapping

1	2	3	4	
A	B	X	D	…
			head	

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				D k_1 *end*	

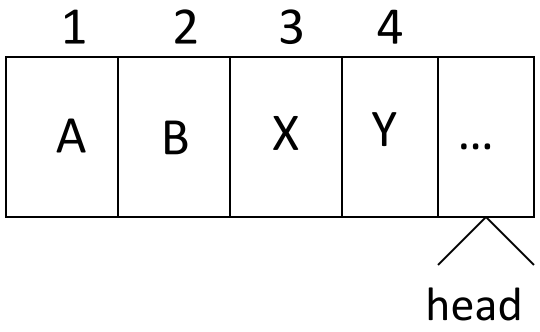
After $\delta(k, \text{C}) = (k_1, \text{X}, \text{R})$ where k is the current state and k_1 the next state

Command Mapping

- $\delta(k, \text{C}) = (k_1, \text{X}, \text{R})$ at intermediate becomes

```
command c_{k,C}(s_3, s_4)
if own in A[s_3, s_4] and k in A[s_3, s_3] and C in A[s_3, s_3]
then
  delete k from A[s_3, s_3];
  delete C from A[s_3, s_3];
  enter X into A[s_3, s_3];
  enter k_1 into A[s_4, s_4];
end
```

Mapping



	s_1	s_2	s_3	s_4	s_5
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				Y	*own*
s_5					b k_2 end

After $\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$ where k_1 is the current state and k_2 the next state

Command Mapping

- $\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$ at end becomes

```
command crightmost_{k,C}(s_4,s_5)
if end in A[s_4,s_4] and k_1 in A[s_4,s_4]
        and D in A[s_4,s_4]
then
  delete end from A[s_4,s_4];
  delete k_1 from A[s_4,s_4];
  delete D from A[s_4,s_4];
  enter Y into A[s_4,s_4];
  create subject s_5;
  enter own into A[s_4,s_5];
  enter end into A[s_5,s_5];
  enter k_2 into A[s_5,s_5];
end
```

Rest of Proof

- Protection system exactly simulates a TM
  - Exactly 1 *end* right in ACM
  - 1 right in entries corresponding to state
  - Thus, at most 1 applicable command
- If TM enters state q_f, then right has leaked
- If safety question decidable, then represent TM as above and determine if q_f leaks
  - Implies halting problem decidable, which we know is false
- Conclusion: safety question undecidable

Other Results

- Set of unsafe systems is recursively enumerable
- Remove **create** primitive; then safety question is complete in **P-SPACE**
- Remove **destroy**, **delete** primitives; then safety question is undecidable
    - Systems are called "monotonic"
- Safety question for biconditional protection systems is decidable
- Safety question for monoconditional, monotonic protection systems is decidable
- Safety question for monoconditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.