ECS 235B Module 40
Nondeducibility

Nondeducibility

- Noninterference: do state transitions caused by high level commands interfere with sequences of state transitions caused by low level commands?
- Really case about inputs and outputs:
    - Can low level subject deduce *anything* about high level outputs from a set of low level outputs?

Example: 2-Bit System

- *High* operations change only *High* bit
  - Similar for *Low*
- σ_0 = (0, 0)
- Sequence of commands:
  - (Heidi, *xor1*), (Lara, *xor0*), (Lara, *xor1*), (Lara, *xor0*), (Heidi, *xor1*), (Lara, *xor0*)
  - Both bits output after each command
- Output is: 00101011110101

Security

- Not noninterference-secure w.r.t. Lara
  - Lara sees output as 0001111
  - Delete *High* outputs and she sees 00111
- But Lara still cannot deduce the commands deleted
  - Don’t affect values; only lengths
- So it is deducibly secure
  - Lara can’t deduce the commands Heidi gave

Event System

- 4-tuple (E, I, O, T)
  - E set of events
  - $I \subseteq E$ set of input events
  - $O \subseteq E$ set of output events
  - T set of all finite sequences of events legal within system
- E partitioned into H, L
  - H set of *High* events
  - L set of *Low* events

More Events …

- $H \cap I$ set of *High* inputs
- $H \cap O$ set of *High* outputs
- $L \cap I$ set of *Low* inputs
- $L \cap O$ set of *Low* outputs
- T_{Low} set of all possible sequences of *Low* events that are legal within system
- $\pi_L: T \rightarrow T_{Low}$ projection function deleting all *High* inputs from trace
  - *Low* observer should not be able to deduce anything about *High* inputs from trace $t_{Low} \in T_{low}$

Deducibly Secure

- System deducibly secure if for all traces $t_{Low} \in T_{Low}$, the corresponding set of high level traces contains every possible trace $t \in T$ for which $\pi_L(t) = t_{Low}$
  - Given any t_{Low}, the trace $t \in T$ producing that t_{Low} is equally likely to be *any* trace with $\pi_L(t) = t_{Low}$

Example: 2-Bit Machine

- Let *xor0*, *xor1* apply to both bits, and both bits output after each command
- Initial state: (0, 1)
- Inputs: $1_H 0_L 1_L 0_H 1_L 0_L$
- Outputs: 10 10 01 01 10 10
- Lara (at *Low*) sees: 001100
  - Does not know initial state, so does not know first input; but can deduce fourth input is 0
- Not deducibly secure

Example: 2-Bit Machine

- Now *xor0*, *xor1* apply only to state bit with same level as user
- Inputs: $1_H 0_L 1_L 0_H 1_L 0_L$
- Outputs: 1011111011
- Lara sees: 01101
- She cannot deduce *anything* about input
  - Could be $0_H 0_L 1_L 0_H 1_L 0_L$ or $0_L 1_H 1_L 0_H 1_L 0_L$ for example
- Deducibly secure

Security of Composition

- In general: deducibly secure systems not composable
- *Strong noninterference*: deducible security + requirement that no *High* output occurs unless caused by a *High* input
  - Systems meeting this property *are* composable

Example

- 2-bit machine done earlier does not exhibit strong noninterference
    - Because it puts out *High* bit even when there is no *High* input
- Modify machine to output only state bit at level of latest input
    - *Now* it exhibits strong noninterference

Problem

- Too restrictive; it bans some systems that are *obviously* secure
- Example: System *upgrade* reads *Low* inputs, outputs those bits at *High*
    - Clearly deducibly secure: low level user sees no outputs
    - Clearly does not exhibit strong noninterference, as no high level inputs!

Remove Determinism

- Previous assumption
  - Input, output synchronous
  - Output depends only on commands triggered by input
    - Sometimes absorbed into commands …
  - Input processed one datum at a time
- Not realistic
  - In real systems, lots of asynchronous events

Generalized Noninterference

- Nondeterministic systems meeting noninterference property meet *generalized noninterference-secure property*
  - More robust than nondeducible security because minor changes in assumptions affect whether system is nondeducibly secure

Example

- System with *High* Holly, *Low* Lucy, text file at *High*
  - File fixed size, symbol ✧ marks empty space
  - Holly can edit file, Lucy can run this program:

```
while true do begin
     n := read_integer_from_user;
     if n > file_length or char_in_file[n] = ✧ then
            print random_character;
     else
            print char_in_file[n];
end;
```

Security of System

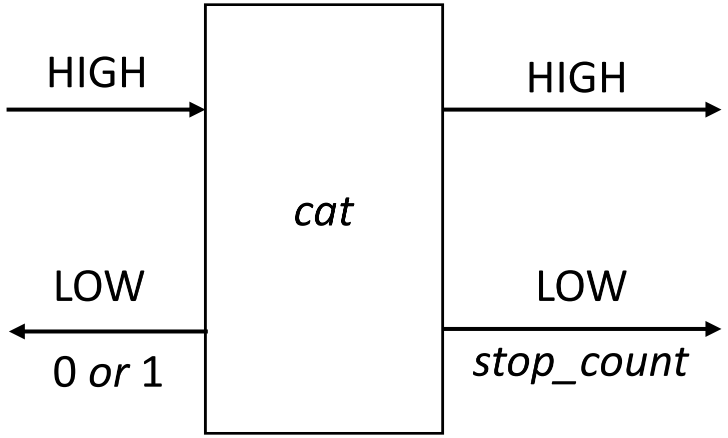
- Not noninterference-secure
  - High level inputs—Holly’s changes—affect low level outputs
- *May* be deducibly secure
  - Can Lucy deduce contents of file from program?
  - If output meaningful (“This is right”) or close (“Thes is riqht”), yes
  - Otherwise, no
- So deducibly secure depends on which inferences are allowed

Composition of Systems

- Does composing systems meeting generalized noninterference-secure property give you a system that also meets this property?
- Define two systems (*cat*, *dog*)
- Compose them

First System: *cat*

- Inputs, outputs can go left or right
- After some number of inputs, *cat* sends two outputs
  - First *stop_count*
  - Second parity of *High* inputs, outputs

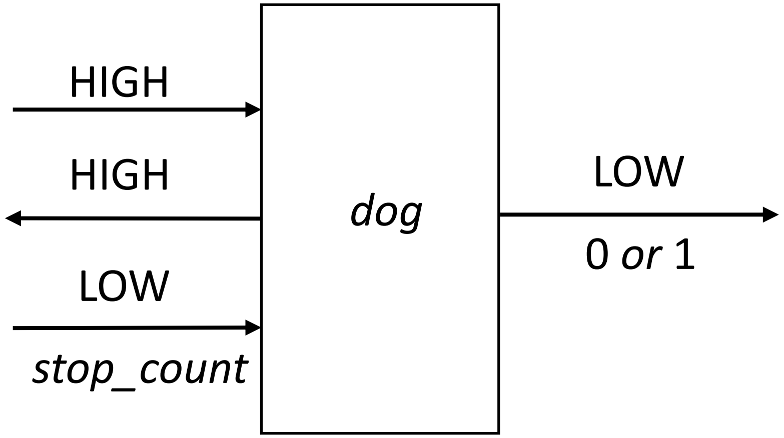


Noninterference-Secure?

- If even number of *High* inputs, output could be:
  - 0 (even number of outputs)
  - 1 (odd number of outputs)
- If odd number of *High* inputs, output could be:
  - 0 (odd number of outputs)
  - 1 (even number of outputs)
- High level inputs do not affect output
  - So noninterference-secure

Second System: *dog*

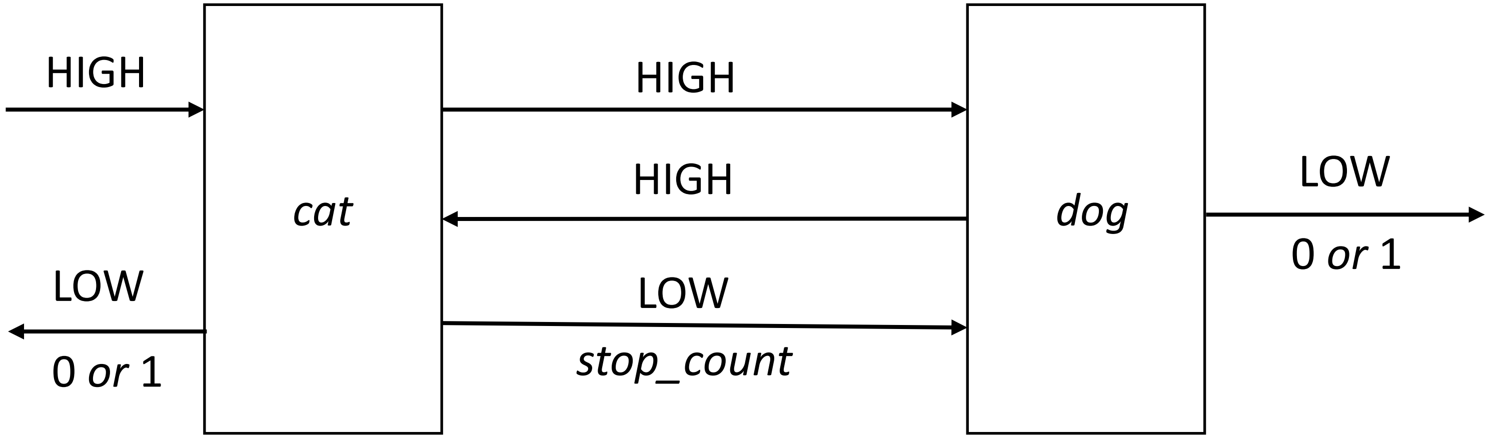
- High outputs to left
- Low outputs of 0 or 1 to right
- *stop_count* input from the left
  - When it arrives, *dog* emits 0 or 1



Noninterference-Secure?

- When *stop_count* arrives:
  - May or may not be inputs for which there are no corresponding outputs
  - Parity of *High* inputs, outputs can be odd or even
  - Hence *dog* emits 0 or 1
- High level inputs do not affect low level outputs
  - So noninterference-secure

Compose Them



- Once sent, message arrives
  - But *stop_count* may arrive before all inputs have generated corresponding outputs
  - If so, even number of *High* inputs and outputs on *cat*, but odd number on *dog*
- Four cases arise

The Cases

- *cat*, odd number of inputs, outputs; *dog*, even number of inputs, odd number of outputs
  - Input message from *cat* not arrived at *dog*, contradicting assumption
- *cat*, even number of inputs, outputs; *dog*, odd number of inputs, even number of outputs
  - Input message from *dog* not arrived at *cat*, contradicting assumption

The Cases

- cat, odd number of inputs, outputs; dog, odd number of inputs, even number of outputs
    - dog sent even number of outputs to cat, so cat has had at least one input from left
- cat, even number of inputs, outputs; dog, even number of inputs, odd number of outputs
    - dog sent odd number of outputs to cat, so cat has had at least one input from left

The Conclusion

- Composite system *catdog* emits 0 to left, 1 to right (or 1 to left, 0 to right)
    - Must have received at least one input from left
- Composite system *catdog* emits 0 to left, 0 to right (or 1 to left, 1 to right)
    - Could not have received any from left (i.e., no HIGH inputs)
- So, *High* inputs affect *Low* outputs
    - Not noninterference-secure