# ECS 289M Lecture 3

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#### Overview

- Safety Question
- HRU Model
- Take-Grant Protection Model



# **Mono-Operational Commands**

- Answer: yes
- Sketch of proof:

Consider minimal sequence of commands  $c_1$ , ...,  $c_k$  to leak the right.

- Can omit delete, destroy

- Can merge all creates into one

Worst case: insert every right into every entry; with *s* subjects and *o* objects initially, and *n* rights, upper bound is  $k \le n(s+1)(o+1)$ 

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### **General Case**

- Answer: no
- Sketch of proof:

Reduce halting problem to safety problem Turing Machine review:

- Infinite tape in one direction
- States K, symbols M; distinguished blank b
- Transition function  $\delta(k, m) = (k', m', L)$  means in state k, symbol m on tape location replaced by symbol m', head moves to left one square, and enters state k'
- Halting state is  $q_{f}$ . TM halts when it enters this state



### **Command Mapping**

![](_page_4_Figure_1.jpeg)

and Information Security

### **Command Mapping**

```
\delta(k_1, D) = (k_2, Y, R) \text{ at end becomes}
command crightmost<sub>k,c</sub>(s_4, s_5)
if end in A[s_4, s_4] and k_1 in A[s_4, s_4]
and D in A[s_4, s_4]
then
delete end from A[s_4, s_4];
create subject s_5;
enter own into A[s_4, s_5];
enter end into A[s_5, s_5];
delete k_1 from A[s_4, s_4];
enter Y into A[s_4, s_4];
enter k_2 into A[s_5, s_5];
end
```

```
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```

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# **Rest of Proof**

- Protection system exactly simulates a TM
  - Exactly 1 end right in ACM
  - 1 right in entries corresponds to state
  - Thus, at most 1 applicable command
- If TM enters state  $q_{f}$ , then right has leaked
- If safety question decidable, then represent TM as above and determine if q<sub>f</sub> leaks
   Implies halting problem decidable
- Conclusion: safety question undecidable

![](_page_6_Figure_0.jpeg)

### Take-Grant Protection Model

- A specific (not generic) system
   Set of rules for state transitions
- Safety decidable, and in time linear with the size of the system
- Goal: find conditions under which rights can be transferred from one entity to another in the system

![](_page_7_Figure_0.jpeg)

![](_page_8_Figure_0.jpeg)

# Islands

- *tg*-path: path of distinct vertices connected by edges labeled *t* or *g* – Call them "tg-connected"
- island: maximal *tg*-connected subjectonly subgraph
  - Any right one vertex has can be shared with any other vertex

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### Initial, Terminal Spans

- *initial span* from **x** to **y** 
  - x subject
  - *tg*-path between **x**, **y** with word in {  $t^*g$  }  $\exists \forall \{x\}$
  - Means  $\boldsymbol{x}$  can give rights it has to  $\boldsymbol{y}$
- terminal span from **x** to **y** 
  - x subject
  - *tg*-path between **x**, **y** with word in {  $t^*$  }  $\cup$  { v }
  - Means x can acquire any rights y has →

![](_page_10_Figure_0.jpeg)

### can•share Predicate

Definition:

 can•share(r, x, y, G<sub>0</sub>) if, and only if, there is a sequence of protection graphs G<sub>0</sub>, ..., G<sub>n</sub> such that G<sub>0</sub> |-\* G<sub>n</sub> using only *de jure* rules and in G<sub>n</sub> there is an edge from x to y labeled *r*.

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#### can•share Theorem

- can•share(r, x, y, G<sub>0</sub>) if, and only if, there is an edge from x to y labeled r in G<sub>0</sub>, or the following hold simultaneously:
  - There is an **s** in  $G_0$  with an **s**-to-**y** edge labeled r
  - There is a subject x' = x or initially spans to x
  - There is a subject s' = s or terminally spans to s
  - There are islands  $I_1, ..., I_k$  connected by bridges, and **x**' in  $I_1$  and **s**' in  $I_k$

![](_page_12_Figure_0.jpeg)

### **Example Interpretation**

- ACM is generic
  - Can be applied in any situation
- Take-Grant has specific rules, rights
  - Can be applied in situations matching rules, rights
- Question: what states can evolve from a system that is modeled using the Take-Grant Model?

# Take-Grant Generated Systems

- Theorem: G<sub>0</sub> protection graph with 1 vertex, no edges; R set of rights. Then G<sub>0</sub> |-\* G iff:
  - G finite directed graph consisting of subjects, objects, edges
  - Edges labeled from nonempty subsets of R
  - At least one vertex in *G* has no incoming edges

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# **Outline of Proof**

- $\Rightarrow$ : By construction; *G* final graph in theorem
  - Let  $\mathbf{x}_1, \ldots, \mathbf{x}_n$  be subjects in G
  - Let  $\mathbf{x}_1$  have no incoming edges
- Now construct *G*′ as follows:
  - 1. Do " $\mathbf{x}_1$  creates ( $\alpha \cup \{g\}$  to) new subject  $\mathbf{x}_i$ "
  - 2. For all  $(\mathbf{x}_i, \mathbf{x}_j)$  where  $\mathbf{x}_i$  has a rights over  $\mathbf{x}_j$ , do " $\mathbf{x}_1$  grants ( $\alpha$  to  $\mathbf{x}_j$ ) to  $\mathbf{x}_i$ "
  - 3. Let  $\beta$  be rights  $\mathbf{x}_i$  has over  $\mathbf{x}_j$  in G. Do " $\mathbf{x}_i$  removes (( $\alpha \cup \{g\} \beta$  to)  $\mathbf{x}_i$ "
- Now G' is desired G

![](_page_14_Figure_0.jpeg)