

Developing a Recursive Program: Listing Permutations

Step #1: Goal and General Algorithm Idea

Scenario: A number of people each need a unique PIN of length n , made up of the digits $1 \dots n$.

Goal: Write a program that generates all possible PINs of length n , made up of the digits $1 \dots n$.

Subgoal: Write a program to generate all permutations of the digits $1 \dots n$.

Let's begin by looking at the permutations of the digits 1, 2, and 3:

```
1 2 3   2 1 3   3 1 2
1 3 2   2 3 1   3 2 1
```

Notice a pattern here: pick the first digit 1, permute the other two, and prepend the 1; then pick the second digit 2, permute the other two, and prepend the 2; and finally, pick the third digit 3, permute the other two, and prepend the 3. More generally, we pick the i th digit, permute all the others, and then prepend that i th digit.

This algorithm suggests recursion. It has a base case, where the recursion stops. Specifically, the permutation of 0 digits is empty, and the permutation of 1 digit is that digit itself. And it has an induction step, namely permuting all but the i th digit and then prepending that.

Now that we have the general idea, let's design the program.

Step #2: Data Representation and Program Structure

Part #1: Data Structures:

Represent the sequence of digits as a list; so the sequence 1, 2, 3 would be treated as a list **L**.

Represent each permutation as an element of another list **I**.

Part #2: Functions

And now we write the function suggested by the above. Let's call it:

function perm(**L**) → returns list of permutations of elements of **L**

First, the base case, when there is no recursion and a value is simply returned. This should happen when the list **L** contains exactly 1 element. We can also add an error check. **L** should never be the empty list, but we can easily check, and so we do:

```
if length of L is 0:
    return empty list
if length of L is 1:
    return list containing L
```

Next, we have to create the list **I** for the list of permutations. Initially, it's empty:

```
I is empty
```

Now for the recursion. We want to loop through **L**, extracting the elements successively. After each extraction, we create a new list without it but with all the other elements. We then permute that list, prepend the extracted element, and continue until we are done with the list:

```
for each element in L:
    remove that element (call it L[i])
    rest of list is L[0 up to i] + L[everything after i]; call this R

    for each element in perm(R):
        prepend L[i]; call the result P
        append P to I
```

Now we have the list of permutations in **I**. So we return it.

```
return I
```

And that's it!

Step #3: Put It into Python

We can translate the function above almost line for line:

```
def perm(L):  
  
    # base cases: if list is empty or  
    # has 1 element, return it as a list  
    # so it can be appended to the list  
    # of permutations  
    if len(L) == 0:  
        return [ ]  
    if len(L) == 1:  
        return [ L ]  
  
    # this will hold the permuted lists of L  
    I = [ ]  
  
    # move each element in the list to the front  
    # and permute the rest of the list; for each  
    # permutation, prepend the front element and  
    # save the result in the list of permutations  
    for i in range(len(L)):  
  
        # drop the i-th element; this gives you  
        # the rest of the list to be permuted  
        R = L[:i] + L[i+1:]  
  
        # generate the permutations of the rest  
        # for each permutation, prepend the one  
        # you held back and add it to the list of  
        # permutations  
        for e in perm(R):  
            P = [ L[i] ] + e  
            I.append(P)  
  
    # return the list of permutations  
    return I
```

Step #4: The Program

To print the permutations, we just print all the elements in the list that *perm* returns:

```
# this is the data to permute  
# here, it's numbers, but it can be anything  
data = [ 1, 2, 3, 4 ]  
  
# get the list of permutations and print it  
for p in perm(data):  
    print(p)
```